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Night Sky Radiations from the Upper Atmosphere*

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IN attempting to speak in honor of Dr. Henry Crew I thought of trying to find a subject about which I might perhaps know nearly as much as he. However, I soon gave this up as impossible, and decided on a short survey of the semiastronomical subject of the light of the night sky. Dr. Crew was astronomer at the Lick Observatory from 1891 to 1892 and for many years was an editor of the *Astrophysical Journal*. Therefore, he is familiar with the night sky and will be appreciative of the efforts of the next two generations to learn about it.

In places remote from artificial illumination at night with no moon the luminosity of the sky is due to several sources of light, all of which are at a considerable distance. The sources are (a) stars and nebulae in interstellar space; (b) zodiacal light, comets, possibly scattered sunlight (if there is something there to scatter the sunlight) in interplanetary space; and (c) emissions from the gases of the upper atmosphere and polar aurorae in the upper atmosphere. Omitting polar aurora from consideration, the high atmospheric emissions are the strongest source of night illumination, being four or five times as intense as all of the other sources combined.

In more detail, the intensity is divided as follows: for the photographic spectrum[1] region 3700–4500A, about $\frac{1}{6}$ due to the stars, $\frac{4}{6}$ due to the high atmosphere emissions, and possibly $\frac{1}{6}$ from interplanetary space (the amount attributed to sources in interplanetary space has decreased as the measurements have improved; it is now down to about $\frac{1}{6}$ of the whole and may eventually become a few hundredths or even disappear); for the visible spectrum[2] seen with the dark-adapted eye, about $\frac{1}{5}$ due to the stars and $\frac{4}{5}$ due to the high atmosphere.

Spectrum

The night sky light is feeble and even with spectrographs of the highest light-gathering power relatively low dispersion and long exposures are necessary to obtain spectra. We need to refer only to recent work and mention that the best spectra at present available are those obtained by Barbier[3] in 1942–44 at the Observatory of Haute Provence, France, and by Elvey, Swings, and Linke[4] in 1939 at the McDonald Observatory, University of Texas.

The dispersion of the spectrograph used by Barbier was 150 and 630A mm^{-1} at 3200 and 5000A, and the length of the spectrum from 3200 to 5000A was about 7 mm; exposures from 100 to 200 hours were used. The McDonald spectro-

* Dr. Henry Crew, Senior Past President of the American Physical Society, was guest of honor at a dinner of the Society on May 17, 1949 during its Semi-Centennial Meeting in Cambridge, Mass. Dr. E. O. Hulburt's paper was presented on the morning following.

[1] D. Barbier, *Comptes Rendus* 224, 635 (1947).
[2] E. O. Hulburt, *J. Opt. Soc. Am.* 39, 211–215 (1949).
[3] D. Barbier, *Ann. d. Geophysik* 1, 224–232 (1945).
[4] Elvey, Swings, and Linke, *Astrophysical J.* 93, 337–348 (1941).

graph had slightly higher dispersion and much greater light-gathering power; the dispersion was 115 and 500A mm^{-1} at 3200 and 5000A, the length of the spectrum from 3200 to 5000A was about 10 mm, and exposures of 9 hours were used.

The night sky spectrum is very complex and is composed of many lines and bands, for the most part incompletely resolved. In Fig. 1 is shown a spectrogram of the night sky taken with the McDonald spectrograph.[4] For wavelengths below 5000A the intensity is rather uniform with no emissions of outstanding strength. Above 5000A the green line 5577, the yellow pair 5890, 5896, and the red group at 6300, 6364, are conspicuous features on the photographic spectrum. The dispersion of present spectra in the red and near infra-red is very low and the region is very poorly resolved. A. B. Meinel[5] of the Lick Observatory is beginning to obtain better data for wavelengths greater than 6000 by the use of a 7500 line transmission grating and an $f/1$ camera, with a dispersion of 180A mm^{-1}; he has published only a few preliminary results.

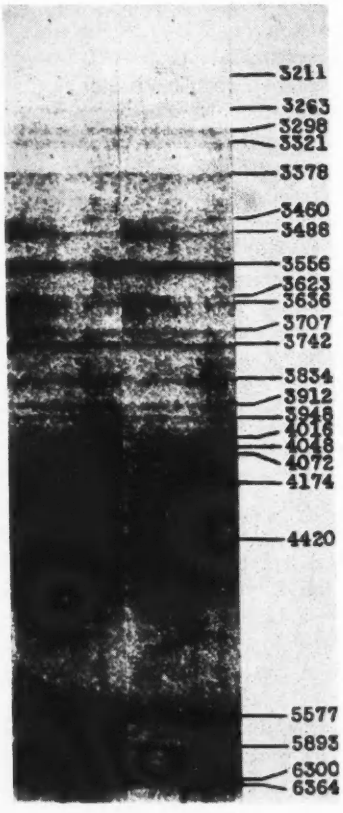

FIG. 1. The spectrum of the night sky, McDonald Observatory (Ref. 4).

The main features of the spectrum of the night sky light, which are considered certain, are:

Atomic lines. The atomic spectrum consists of the lines 5577, 6363.9, and 6300.2, forbidden transitions of oxygen, with 5896 and 5890, sodium.

Molecular lines and bands. Among the most intense are 10,440 and 6560, 1st positive system of N_2; the 10,440 emission is by far the strongest emission thus far observed, being about 30 times as intense as the green line 5577, the next most intense emission. It was discovered in 1940 by Stebbins, Whitford, and Swings,[6] by means of a photoelectric cell and filters, who determined the wavelength within ±25A, and independently by Herman, Herman, and Gauzit.[7]

The strongest blue-violet bands consist of the Vegard-Kaplan system of N_2; about 35 bands are identified.

The strongest ultraviolet bands are the Herzberg system of O_2, about 20 bands are identified.

Some weaker bands occur in violet and ultraviolet regions of the spectrum due to unknown systems.

A number of unidentified lines or bands, some partially resolved, occur from 5000 to 10,000A.

A sort of continuum and unresolved bands are found, which are fairly weak in the ultraviolet and which increase in intensity for wavelengths longer than 4000A.

A weak continuous background with trace of Fraunhofer absorptions which was mentioned in earlier work was not observed in the better more recent spectrograms.

[5] A. B. Meinel, *Pub. Ast. Soc. Pacific* 60, 373–377 (1948).
[6] Stebbins, Whitford, and Swings, *Astrophysical J.* 101, 39–46 (1945).
[7] Herman, Herman, and Gauzit, *J. de Physique*, Ser. 8 6, 182 (1945).

The Schumann-Runge system of O_2 and the Lyman-Birge system of N_2 have been looked for, but no certain identifications have been made; similarly for CO and CO_2, ozone, oxides of nitrogen, NH_2 and N_3, CH, CN, OH, H_2O, and compounds of Na, Si, and S.

The spectral intensity distribution received at the surface of the earth is approximately as follows:[8]

Wavelength	Emitter	Intensity
10440A	N_2	190×10^{-4} erg cm^{-2} sec^{-1}
6300, 6363	O	5×10^{-4}
5890, 5896	Na	0.7×10^{-4}
5577	O	7×10^{-4}
Main band systems	N_2, O_2	10×10^{-4}.

The only reported spectral energy distribution of the night sky light is that of Babcock and Johnson[9] derived from measurements of a small photographic spectrum of dispersion 1100A mm^{-1} at H_γ. Their curve is given in Fig. 2; it rises with increasing wavelength throughout the extent of the spectrum from 3800 to 6500A, with bumps at the various lines and bands. A color temperature of 3450°K was ascribed to it. Thus the night sky light may be said to be yellowish in color.

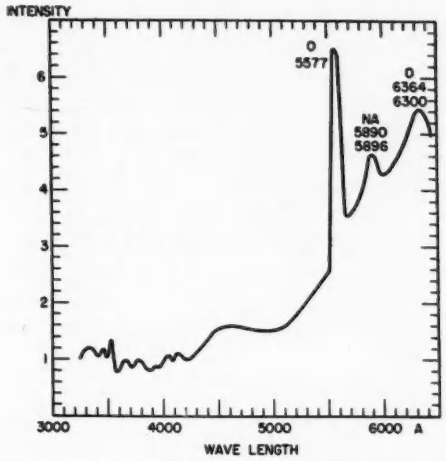

FIG. 2. Spectral distribution of night sky light (Ref. 9).

Variations

Excluding polar aurorae the variations in the total intensity of night sky emission are relatively small, rarely as much as a factor of 2. There appear to be during the full nighttime hours many small irregular and complex intensity variations which vary with the place in the sky and which are different for the different wavelengths. There are strong intensity changes during twilight in certain lines.

Diurnal variations. Dufay and Mao-Lin[10] at Haute Provence Observatory obtained 588 spectrograms during 189 nights from October 1940 to January 1944. From these spectrograms systematic measurements were made of the intensity variations of the oxygen lines 5577 and 6300A and of the sodium pair 5893. The 5577 line usually showed a weak maximum around midnight (not more than 40 percent) often obscured by fluctuations; the 6300 and 5893 lines merely weakened slowly during the night, the enfeeblement being greater for 6300 than that for 5893 which only amounted to 10 or 15 percent.

Twilight effects. The green line 5577 was observed to exhibit no twilight enhancement. On the other hand the sodium yellow lines 5893 dropped suddenly in intensity to 1 percent of their former value at about 10 minutes after sunset[11] with a corresponding recrudescence near dawn. Likewise the red oxygen lines 6300 showed a similar but slower change during twilight.[12] The strong N_2^+ flash at twilight discovered by Slipher[13] is intense in aurorae but absent from the night sky.

Annual variations. In a temperate latitude 5577 has slight maxima in October and February, and a 27-day variation in intensity, hence a faint correlation with solar phenoména; 6300 and 5892 have maxima near midwinter and minima in summer of amplitude factors of about 2.5 and 5, respectively; they seem indifferent to solar phenomena.[10]

Latitude variations. Measurements with calibrated and standardized visual photometers indicated no changes, which could not be attributed to variations in haze, in the visual brightness

[8] D. R. Bates, Gassiot Committee Report, Physical Soc. London, 21–33 (1948).
[9] H. W. Babcock and J. J. Johnson, *Astrophysical J.* **94**, 271–275 (1945).
[10] J. Dufay and T. Mao-Lin, *Ann. d. Geophysik* **2**, 189–230 (1946).
[11] R. Bernard, *Zeits. f. Physik.* **110**, 291–302 (1938).
[12] C. T. Elvey and A. H. Farnsworth, *Astrophysical J.* **96**, 451–467 (1942).
[13] V. M. Slipher, *Roy. Astr. Soc. M.N.* **93**, 657–668 (1933).

of the night sky with latitude when no polar aurorae were visible.[2,14] Figure 3 is a photograph of one of the visual photometers. The observations were made at Bocaiuva (Brazil), Bikini, Maryland, Whiteface Mountain (New York State), and Greenland, at latitudes −17°, +12°, +39°, +45°, and +63°, respectively. Average data for these stations are plotted in Fig. 4. Some older visual measurements[15] of the sky near Polaris indicated that the brightness increased by a factor of 4 from latitudes +44° to +80°, but this result may have been influenced by polar aurorae.

Visual measurements of the night sky luminosity refer mainly to the strong oxygen atomic line 5577. Because, by multiplying the night sky spectral energy curve by the sensitivity curve of the dark-adapted eye, one finds that 5577 has the main effect ($\frac{8}{10}$) with only minor influence of the oxygen red lines 6300, 6363 the sodium yellow lines 5896, 5890 and the molecular bands in the blue and green.

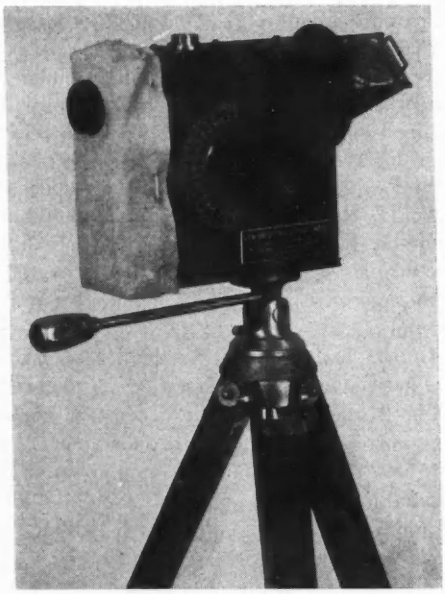

FIG. 3. Low brightness visual photometer, Naval Research Laboratory (Ref. 2).

Variation with zenith angle. The variation of the visual brightness of the night sky with zenith angle was found to be about the same for latitudes from −17° to +63°, and increased from about 130 millimicrolambert (mμL; 1 mμL = 10^{-9} lambert = 2.96×10^{-7} candle per square foot) at the zenith to a maximum of about 250 mμL at about 15° above the horizon for a clear atmosphere; with slight haze the maximum near the horizon became less pronounced, and with more haze disappeared.[2] The variation was observed to be somewhat different for red and green wavelengths,[16] but has not been thoroughly explored for all wavelengths.

Altitude

The altitude of the high atmospheric emissions has been determined from the intensity variation with zenith angle, and erratic and uncertain values from 60 km to several hundred km were found. The uncertainties were due to nonuniformity of the emissions and to incompletely worked-out corrections for atmospheric attenuations. But even with better correction formulas no improved or more trustworthy values of the altitudes were determined;[2] and in no case was the atmospheric attenuation measured at the same time that the sky intensity was observed. Swings[17] stated the situation thus: "There are great irregularities in the distribution of the emission over the sky. These irregularities appear consistently when simultaneous exposures in different azimuths at the same zenith distance are compared. No layer of uniform brightness exists, but rather a set of bright clouds which move around and change in brightness. The only remaining hope is that, by taking a sufficiently large number of observations, the erratic fluctuations will average out."

The altitude of the yellow emissions 5890, 5896 of sodium was determined in another way based on the fact that these radiations decreased suddenly in intensity at twilight when the region in which they originated was shielded from the

[14] Data for latitude +63° were observed in February and March, 1949, by personnel at an Air Force Weather station in Greenland, using an NRL visual low brightness photometer.

[15] B. Fessenkoff, *C.R. (Doklady) Acad. Sci. URSS* **32**, 320–322 (1941).

[16] Abadie, Vassy, and Vassy, *Ann. d. Geophysik* **1**, 189–224 (1944).

[17] P. Swings, *The atmospheres of the earth and planets* (University of Chicago Press, 1948), Chapter 6.

direct ultraviolet rays of the sun. The calculations were made by Penndorf[18] from the observations of Vegard and Tonsberg[19] in February and March, 1949, at Tromso and Oslo. The altitude of the sodium emission at twilight was determined unmistakably to be in the region from 80 to 116 km. At present it seems reasonable to keep in mind the probability that the various night sky radiations may originate in different levels which may change in the course of the night and which may vary with the latitude and season.

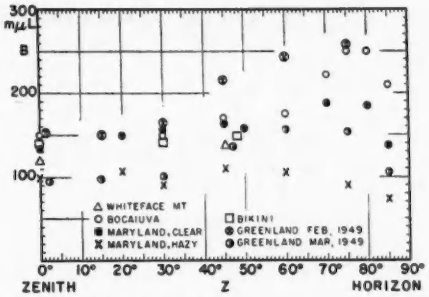

FIG. 4. Night sky brightness against zenith angle at several stations.

Theory

The spectral identifications show that most of the night sky emissions come from the two most abundant gases of the atmosphere, oxygen and nitrogen; a small portion comes from sodium. No one has questioned the general notion that the nocturnal emissions derive their energy from the sun, but no processes or quantitative details have been agreed upon because information is lacking about radiation transitions and atmospheric composition. In fact, without exception, all theoretical processes, qualitative or quantitative, thus far proposed, have been weighted down with a wealth of criticism.[8]

The excitation energy of the line systems 5577 OI, 6300 OI, 5893 NaI are 4.2, 2.0, and 2.1 ev, respectively; and of the band systems Vegard-Kaplan N_2, Herzberg O_2, Schumann-Runge O_2, and Lyman N_2 are 7.0, 4.7, 6.2, and 8.7 ev, respectively. It has been suggested that the energy is provided by (a) the association energies of atomic O and N, which are 5.09 and 7.38 ev, respectively; (b) the energies of ionization, which are about 13.5 and 14.5 ev for the first ionization potentials of O and N, respectively; (c) particles of solar origin. It appeared that (a) was sufficient for the line systems but for none of the band systems except Herzberg; (b) seemed energetically satisfactory, but not completely in accord with the Frank-Condon probabilities; (c) is speculative and has not been put in quantitative terms.

The relatively small amount of sodium in the upper atmosphere necessary to account for the 5893 emissions may come either from the surface of the earth, as the sea, or from interplanetary space. Both views have been suggested and the correct one is not known.

In conclusion, we have attempted to exclude the aurora from the foregoing survey of the night sky light knowing that this cannot be done, and in the end should not be done. It is probable that a complete understanding of the night sky cannot be reached without at the same time having a complete understanding of the aurora. Both, it is believed, derive their primal energy from the sun. The auroral emissions are from the same familiar gases of our atmosphere, oxygen and nitrogen, as are those of the night sky, but with different spectral distributions, with enormously greater intensities and in quite different geometrical patterns. The auroral energy is received in concentrated form from explosions or outbursts on the sun, in contrast to the relatively gentle and steady solar energy stream which supports the normal night sky light. The kinship between the aurora and the night sky light may be analogous to that between a hurricane and a gentle rain.

[18] R. Penndorf, *J. of Meteor.* **5**, 152–160 (1948).
[19] L. Vegard and E. Tonsberg, *Geophysical Publ.* **13**, 1–22 (1940).

History is simply a piece of paper covered with print; the main thing is still to make history, not to write it.—BISMARCK.

On the Mechanism of Brownian Motion in Liquids

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1. Einstein's Law

THE following derivation of the well-known formula for the mean square path traveled in Brownian motion by a spherical particle suspended in a liquid has the advantage of being more illuminating than the usual derivations.[1] In fact, it is based upon the qualitative discussion used in many introductory courses of modern physics, here made quantitative by a few simple formulas.

Let a spherical particle of radius r, mass m, be suspended in a liquid of viscosity η. Occasionally a molecule may hit the particle with exceptional vigor and set the particle into motion through the liquid. As a consequence, the probability of head-on collisions against other molecules is slightly increased, while the probability of a collision by a molecule overtaking the particle is slightly decreased. Also, the particle will, in subsequent collisions, give back some of its gained momentum to the newly colliding molecules.[2] Altogether, there are correlations between the effects of preceding collisions and of subsequent collisions. These correlations favor a retardation of the particle in its motion, an effect known as *friction*. From the above it is easily seen that in first approximation friction may be described as a force proportional to the velocity of the particle, so that

$$m d\mathbf{v}/dt = -S\mathbf{v}. \tag{1}$$

For big spheres one may use Stokes' law, putting

$$S = 6\pi\eta r, \tag{2}$$

but it is well known that this law breaks down for spheres of very small radius.[3] As we shall see below, it breaks down sooner in liquids of high viscosity than in liquids of low viscosity.

Having taken into account the correlations between subsequent collisions in this way, we may further consider them as independent of each other.[4] At random times t_i they will give the particle random impulses $m\mathbf{V}_i$. Due to these random forces as well as to the friction, the total velocity $\mathbf{v}$ of the particle at a time t will be the vector sum of the velocities $\mathbf{v}_i$ left over from $\mathbf{V}_i$ after an interval $(t-t_i)$. Hence

$$\mathbf{v}(t) = \sum_{(i)} \mathbf{v}_i(t); \quad v_{ix}(t) = V_{ix} e^{-(S/m)(t-t_i)}, \tag{3}$$

where $v_{ix}(t)$ denotes the x-component of $\mathbf{v}_i(t)$ and where the t's between brackets indicate that the velocities are functions of t. The summation is to be extended over all collisions (numbered by the label i) prior to the time t.

When we calculate the average of the square $[\mathbf{v}(t)]^2$ of the velocity at a time t, and we take $\mathbf{v}(t)$ from the first part of Eq. (3), then all terms $(\mathbf{v}_i \cdot \mathbf{v}_j)$ for $i \neq j$ can be omitted, being equally often negative as positive. Taking also the time average of this mean square velocity over some long time interval from 0 to t_0, we find, say for the mean square of the x-component

$$\{[v_x(t)]^2\}_{\text{av}} = t_0^{-1}\Big\{ \sum_{(i)\,(0<t_i<t_0)} \int_{t_i}^{t_0} [v_{ix}(t)]^2 dt + \sum_{(i)\,(t_i<0)} \int_0^{t_0} [v_{ix}(t)]^2 dt \Big\}. \tag{4}$$

We substitute here $v_{ix}(t)$ from the second part of Eq. (3), and perform the integrations. Assuming that there are f collisions per second giving appreciable impulses to the particle, we may further replace the summation signs by their

[1] A derivation showing similarity to the one given here can be found in Müller-Pouillet's *Lehrbuch der Physik*, vol. III/2. 11. Auflage, pp. 327–328. (F. Vieweg & Sohn, Braunschweig, 1925.) There, however, the relaxation time $\tau = m/S$ replaces the long time t_0 used in our derivation, and as a consequence in Eq. (353), *loc. cit.*, a factor remains unexplained, which does not appear in our derivation. Moreover, relaxation time is assumed there to be of the order of magnitude of the time between two collisions effectively disturbing the rectilinearity of the path of the particle, quite contrary to our result expressed immediately following our Eq. (24).

[2] I wish to thank Dr. H. M. James for valuable discussions about these and some other statistical questions.

[3] Müller-Pouillet, see reference 1, pp. 83–89.

[4] That is, we describe the forces exerted by molecules against a moving sphere as the superposition of the force of friction described by Eq. (1) and independent completely random forces giving the particle sudden impulses at irregular intervals.

equivalents

$$\sum_{(0<t_i<t_0)}{}^{(i)} = f\int_0^{t_0} dt_i; \quad \sum_{(t_i<0)}{}^{(i)} = f\int_{-\infty}^{0} dt_i. \tag{5}$$

Performing these last integrations too, while we replace $V_{ix}{}^2$ by its average, we obtain

$$\{[v_x(t)]^2\}_{\text{av}} = fm\langle V_{ix}{}^2\rangle_{\text{av}}/2S, \tag{6}$$

a result which we might also have obtained with less integrating but more thinking[5] from

$$\langle v_x(t)^2\rangle_{\text{av}} = (ft_0)\times\left\{t_0{}^{-1}\int_{t_i}^{\infty}[v_{ix}(t)]^2dt\right\}. \tag{7}$$

[5] If $v_{i(-nt_0)x}(t)$ is a symbolic notation for the x-component of the velocity at t left over from the velocity acquired by a collision approximately at the time (t_i-nt_0) (that is, from a collision that took place about nt_0 seconds before collision number i), then in Eq. (4) we might replace the second sum (over $t_i<0$) by

$$\sum_{(0<t_i<t_0)}{}^{(i)}\ \sum_{n=1}^{\infty}\int_0^{t_0}[v_{i(-nt_0)x}(t)]^2dt,$$

since obviously for each collision taking place before $t=0$ there will be some other collision taking place between $t=0$ and $t=t_0$ just about a multiple of t_0 seconds later. Now, for purposes of calculating the mean square, we may write $v_{ix}(t+nt_0)$ instead of $v_{i(-nt_0)x}(t)$, because the mean square of $v_{ix}(t')$ will depend on the time difference $(t'-t_i)$ only. Also, we may write

$$\sum_{n=1}^{\infty}\int_0^{t_0}[v_{ix}(t+nt_0)]^2dt = \sum_{n=1}^{\infty}\int_{nt_0}^{(n+1)t_0} v_{ix}(t')^2dt' = \int_{t_0}^{\infty} v_{ix}(t')^2dt'.$$

Combining the result thus obtained with the first sum in Eq. (4) we find that

$$\{[v_x(t)]^2\}_{\text{av}} = t_0{}^{-1}\sum_{0<t_i<t_0}{}^{(i)}\int_{t_i}^{\infty}[v_{ix}(t)]^2dt.$$

As the integral over t here can be written as an integral over $(t-t_i)$ and then becomes clearly independent of i, we may now replace the summation over i simply by a multiplication by the total number of collisions ft_0 in the time interval from 0 to t_0, and thus obtain the result of Eq. (7). Thus, the extension of the integration over t in Eq. (7) to ∞ (instead of to t_0 as in Eq. (4)) here accurately takes care of the after-effects from all the collisions at times before $t=0$, which in Eq. (4) were represented by the second sum, but which in Eq. (7) do not appear explicitly. For the rest, instead of taking these after-effects of all those collisions in the far past accurately into account, we might as well have neglected them completely, as well as we might have written integrals from t_i to ∞ for the original integrals from t_i to t_0 simply by neglecting the difference between these integrals, at least, if we choose $t_0\gg m/S$. Thus, we might as well replace the words "more thinking" in the text by "more neglecting," and then have by Eq. (7) a simple means of obtaining the result shown in Eq. (6), by performing the one integration over t in Eq. (7) after substitution of $v_{ix}(t)$ from Eq. (3) into it.

The law of equipartition of energy for the one degree of freedom v_x now gives

$$\tfrac{1}{2}m\{[v_x(t)]^2\}_{\text{av}} = \tfrac{1}{2}kT, \tag{8}$$

(and similarly for v_y and v_z), so that, by Eq. (6)

$$fm^2\langle V_{ix}{}^2\rangle_{\text{av}} = 2SkT. \tag{9}$$

The total displacement $\mathbf{x}(t_0)$ of the particle during the time interval from $t=0$ to $t=t_0$ is again a vector sum

$$\mathbf{x}(t_0) = \sum_{(t_i<t_0)}{}^{(i)}\mathbf{x}_i(t_0); \quad \mathbf{x}_i(t_0) = \int_0^{t_0}\mathbf{v}_i(t)dt. \tag{10}$$

In the integral, we put $\mathbf{v}_i(t)=0$ for $t<t_i$. We shall calculate the mean square of the x-component of $\mathbf{x}(t_0)$. Again the cross terms $(\mathbf{x}_i\cdot\mathbf{x}_j)$ have a mean value zero, and we find the following result showing some analogy with Eq. (4):

$$\langle x(t_0)^2\rangle_{\text{av}} = \sum_{(0<t_i<t_0)}{}^{(i)}\left\{\int_{t_i}^{t_0}v_{ix}(t)dt\right\}^2 + \sum_{(t_i<0)}{}^{(i)}\left\{\int_0^{t_0}v_{ix}(t)dt\right\}^2. \tag{11}$$

Into this expression, we substitute $v_{ix}(t)$ from Eq. (3), integrate over t and replace $V_{ix}{}^2$ by its average. Next, we make use of Eq. (5). Performing the integrations over t_i and putting $m/S=\tau$, we find

$$\{[x(t_0)]^2\}_{\text{av}} = fm^2\langle V_{ix}{}^2\rangle_{\text{av}}S^{-2}\{t_0-\tau(1-e^{-t_0/\tau})\}. \tag{12}$$

Assuming that t_0 is chosen to be large in comparison with the relaxation time $\tau=(m/S)$, we find

$$\{[x(t_0)]^2\}_{\text{av}} \approx ft_0m^2\langle V_{ix}{}^2\rangle_{\text{av}}/S^2, \tag{13}$$

a result which we might again have obtained in an easier way by more thinking and neglecting[6] and less integrating, by means of

$$\{[x(t_0)]^2\}_{\text{av}} \approx (ft_0)\times\left\{\int_{t_i}^{\infty}v_{ix}(t)dt\right\}^2. \tag{14}$$

[6] If $\tau\ll t_0$, the motion of the particle due to momentum acquired prior to $t=0$ is negligible during the time interval from $t=0$ to $t=t_0$, while the displacements $x_i(t_0)$ due to the collisions during this interval are practically equal to $x_i(\infty)$ with the exception of the displacements due to only very few of these (ft_0) collisions near the very end of this long interval. Thus $\sum_{(i)}\langle x_i(t_0)^2\rangle_{\text{av}}$ leads to Eq. (14).

TABLE I. Critical constants of liquids.

Liquid	Chemical formula	M	ρ g/cm³	ρ_{crit} g/cm³	p_{crit} atmos.	η (at 20°C) poise	$10^{-5}v_s$ cm/sec	$(3\rho_{crit}-\rho)$ g/cm³
Acetone	CH_3COCH_3	58.1	0.792	0.268	47	0.0033	1.25	0.012
Aniline	$C_6H_5NH_2$	93.1	1.022	?	52.4	0.044	1.6	?
Benzene	C_6H_6	78.1	0.8794	0.531	48	0.0065	1.3	0.714
Carbon tetrachloride	CCl_4	153.8	1.595	0.558	45	0.0097	0.93	0.079
Cyclohexanol	$C_6H_{11}OH$	100.2	0.9624	?	?	0.68	1.6	?
Ethyl acetate	$CH_3COOC_2H_5$	88.1	0.901	0.308	37.8	0.0045	1.2	0.023
Ethyl alcohol	CH_3CH_2OH	46.1	0.7893	0.2755	63.1	0.0120	1.2	0.0372
Ethyl ether	$C_2H_5OC_2H_5$	74.1	0.7135	0.2625	35.5	0.0023	1.03	0.074
Glycerin	$CH_2OHCHOHCH_2OH$	92.1	1.260	?	?	14.9	1.9	?
Mercury	Hg	200.6	13.55	4 or 5	>200?	0.0155	1.45	?
Methylene bromide	CH_2Br_2	173.9	2.495	?	?	0.00104	0.97	?
Phenol	C_6H_5OH	94.1	1.072	?	60.5	0.12	?	?
Propylacetate	$CH_3COOC_3H_7$	102.1	0.887	0.296	32.9	0.0059	?	?
Water	H_2O	18.0	1.00	0.4	217.7	0.010	1.45	0.2

Now substituting $fm^2\langle V_{iz}^2\rangle_{av}$ from Eq. (9) into Eq. (13) we obtain Einstein's law for the mean square displacement in the x direction:

$$\{[x(t_0)]^2/t_0\}_{av}=2kT/S\ (=kT/3\pi\eta r). \quad (15)$$

The expression between brackets is valid, if Eq. (2) is true. We may remark that both f and $\langle V_{iz}^2\rangle_{av}$ drop out in reaching Eq. (15). This makes the result independent of special assumptions about the number of collisions and their direct effect upon the particle, as long as we can believe the equipartition law, Eq. (8).

2. Discussion

It is now interesting to estimate at least the orders of magnitude of the impulses mV_i and of their frequency f. An upper limit for the latter is given by the number of thermal collisions per second of molecules against the spherical particle, which is

$$F=4\pi r^2\cdot\tfrac{1}{4}\langle c\rangle_{av}\cdot N'=2N'r^2(2\pi kT/\mu)^{\frac{1}{2}}. \quad (16)$$

Here, $\langle c\rangle_{av}$ is the average thermal velocity of molecules in the liquid; the factor $\frac{1}{4}$ comes in through averaging over angles. N' is a kind of "effective" density of the liquid molecules. If the liquid were a rarified gas, N' would be simply the number of molecules per cm³ ($=N$). In our case, N' is increased due to the crowdedness of the molecules in a liquid, by a factor C, which may be estimated[7] as being of the order of magnitude $\mathfrak{V}/(\mathfrak{V}-b)$, where $\mathfrak{V}$ is the molal volume, while b according to Van der Waals' theory of fluids would be one-third of the critical (molal) volume. Introducing specific gravities ρ instead of molal volumes by $\rho=M/\mathfrak{V}$, we find by this first method of estimating C

$$C_1=\frac{3\rho_{crit}}{3\rho_{crit}-\rho},$$

thence

$$N_1'=C_1N=\frac{3\rho\rho_{crit}}{\mu(3\rho_{crit}-\rho)}. \quad (17:1)$$

Here, μ is the mass of a molecule of the liquid.

A second method of estimating C is given by

$$C_2=\frac{v_s}{\frac{1}{2}\langle c\rangle_{av}},$$

thence

$$N_2'=C_2N=\frac{\rho v_s}{\mu}\left(\frac{\pi\mu}{2kT}\right)^{\frac{1}{2}}. \quad (17:2)$$

This guess is based on the following ideas: According to the well-known formula for the velocity of sound $v_s=(\gamma p/\rho)^{\frac{1}{2}}$ (with $\gamma=c_p/c_v$ for an ideal gas and $\gamma\approx 1$ for a liquid), this velocity would be a constant independent of the frequency ν or the

[7] The occurrence of the factor $(\mathfrak{V}-b)$ instead of $\mathfrak{V}$ in Van der Waals' equation is roughly explained by stating that, because of the finite dimensions of the molecules, they do not have a volume $\mathfrak{V}$ at their disposal for free motion, but a volume that is smaller due to the fact that each molecule has to stay with its center at a distance of at least twice the molecular radius from the centers of other molecules. As a consequence, the molecules hit the walls more frequently and thus exert a larger pressure, which is now proportional to $1/(\mathfrak{V}-b)$ instead of to $1/\mathfrak{V}$, thus being increased by a factor $C_1=\mathfrak{V}/(\mathfrak{V}-b)$. Exactly the same reasoning may be considered applicable to the calculation of the number of collisions against a suspended particle.

wavelength λ of the vibrations. From the derivation of this formula it is clear that v_s is meant here to be the *phase* velocity of the sound waves, so that $\nu = v_s/\lambda$. The *group* velocity of such sound waves then can be calculated from this by the well-known formula $v_g = d\nu/d(1/\lambda)$. But since v_s was constant in our equation for ν, this gives simply $v_g = v_s$. On the other hand, the group velocity is the velocity with which a short signal would be propagated through the liquid. As the wave front of such a signal moves forward through the liquid by the motion of molecules, one would then expect this velocity to be of the same order of magnitude as the average component (in the direction of propagation of the wave) of the thermal velocity c of the molecules. That is, one expects $v_s \approx \frac{1}{2}\langle c\rangle_{av}$. For an ideal gas this is actually the case (with $v_s = (\gamma\pi/2)^{\frac{1}{2}} \cdot \frac{1}{2}\langle c\rangle_{av}$). If the molecules are very crowded, however, the velocity of sound can become much larger than the thermal velocities; as each time, when a molecule at the front of a sound wave collides against another molecule in front of it, the wave motion jumps forward over a distance up to twice the radius of a molecule. This causes an increase of the velocity of sound by the same factor, by which also the frequency of collisions of molecules against the walls is increased by the crowdedness of the molecules. (The reasoning in both cases is almost identical.) The latter ratio was just $C_1 = \mathfrak{V}/(\mathfrak{V}-b)$, so that one might expect now an approximate equality between $C_1 = \mathfrak{V}/(\mathfrak{V}-b)$ and $C_2 = v_s/\frac{1}{2}\langle c\rangle_{av}$.

It must be granted that in practice the agreement between these two expressions for various liquids is far from striking, as is shown in Table II, which has been calculated with use of some of the data listed in Table I. Therefore, in the following, both expressions (17:1) and (17:2) are used from time to time (compare for instance Eq. (26:I1) with Eq. (26:I2)). For many liquids, it is simpler to use Eq. (17:2) than to use Eq. (17:1) for often the velocity of sound is known, whereas the critical density (from which b is estimated) is not known.

A third estimate of N' can be made on the grounds that the frequency F of thermal collisions also determines the internal pressure in a liquid, so that

$$p_{int} = N'kT.$$

TABLE II. Crowdedness factor for liquids.

Liquid	Acetone	Benzene	Carbon tetra-chloride	Ethyl acetate	Ethyl alcohol	Water
C_1	67	2.23	21	40	22.2	6
C_2	7.6	9.2	9.2	9.0	6.5	4.93

By Van der Waals' theory, with $\mathfrak{V} \approx b$ for a liquid, the internal pressure $p_{int} = p + (a/\mathfrak{V}^2) \approx a/\mathfrak{V}^2 \approx a/b^2 = 27p_{crit}$, so that

$$N_3' = 27p_{crit}/kT. \qquad (17:3)$$

Effective molecular densities as calculated by these three completely different methods do not differ among each other more than by factors smaller than 100. For various liquids at room temperature, they all give for (F/r^2), by Eq. (16), values between 10^{27} and 10^{29} sec^{-1} cm^{-2}. (See Table III for the results obtained by use of Eqs. (17:2) and (17:3) respectively.)

If such thermal collisions against the particle occurred one at a time, one could calculate the impulse by the law of conservation of momentum. The impulse $m\mathbf{V}$, which the particle would receive by such an unhindered collision, would be given by $m\mathbf{V} = 2\mu\mathbf{u}$, where $\mathbf{u}$ is the vector component of the thermal molecular velocity $\mathbf{c}$ normal to the surface of the particle. Averaging the squares of these impulses as the collisions take place,[2] we have

$$m^2\langle \mathbf{V}^2\rangle_{av} = 4\mu^2\langle u^2\rangle_{av}.$$

In the calculation of $\langle u^2\rangle_{av}$ it should be remembered here that the frequency of collision itself is proportional to u, so that

$$\langle u^2\rangle_{av} = \left(\int_0^\infty e^{-\mu u^2/2kT} u^2 du\right) \Big/ \left(\int_0^\infty e^{-\mu u^2/2kT} u du\right) = 2kT/\mu.$$

Thence

$$m^2\langle \mathbf{V}^2\rangle_{av} = 8\mu kT; \quad m^2\langle V_x^2\rangle_{av} = 8\mu kT/3. \qquad (18)$$

The momentum acquired by the particle by one such collision is rather small and much less than the total momentum mv of the particle required for validity of the equipartition law, Eq. (8), since the mass μ of one molecule is much smaller than the mass m of the suspended par-

ticle. Yet, these collisions will be rather effective in causing a jerky motion of the particle, due to their high frequency. This frequency, however, is in fact not given by F as calculated by Eqs. (16)–(17), but is much lower. That the frequency of such one-at-a-time collisions, at least for sufficiently big particles, cannot possibly be given by Eq. (16), is easily seen as follows: In Sec. 1, we derived Eq. (9) from the law of equipartition of energy, Eq. (8), together with the law of friction, Eq. (1), without specifying at all what the value of S might be.[8] Now, if collisions, imparting momenta mV_{iz} to the particle as large as the quantity mV_z given by Eq. (18), occurred with a frequency f as large as the quantity F from Eqs. (16)–(17), then Eq. (9) would give us an expression for S proportional to the *square* of r (through the factor $f=F$). For liquids of molecular weight $M\approx 90$ at room temperature (20°C) we should thus find

$$S\approx(10^{23\pm 1}r^2)(4\mu/3)\approx 2\times 10^{6\pm 1}r^2. \qquad (19)$$

This result must obviously be *wrong* for particles that are not very small, because for such particles we know that Stokes' law, Eq. (2), is valid, by which S is proportional to the first power of r only. That is, for large particles the friction is actually *smaller* than one would calculate on the assumption that the number of one-at-a-time collisions per second were as large as the number of thermal collisions F.

The reason why f for not-too-small particles actually is much smaller than F, becomes clear, if we remember how extremely large is the number of thermal collisions F against the particle in a liquid. Even against a particle as small as $r=10^{-5}$ cm we found 10^{17} to 10^{19} collisions per second. (See below Eq. (17:3).) As each collision takes a finite time, this means that very often one collision has not yet been completed before other collisions start. Since the molecules of the liquid have a finite size, those new collisions cannot take place from exactly the same direction, so the average effect of simultaneity of collisions will be a partial neutralization of the effects of the single collisions. This neutralizing effect should not be confused with the friction discussed in the first section. The friction there considered was a consequence of the velocity which the particle had already obtained, while here we discuss an effect that is not due to the velocity of the particle, but which, on the contrary, acts to prevent the motion from starting at all.

Thus we should not try here to use the statistical considerations leading to Eqs. (16)–(18) for a calculation of the friction by kinetic considerations, as one could do in the case of a rarified gas,[9] for which f indeed would be equal to F for small particles. Such a derivation would simply establish statistically Eq. (19), which we obtained above from Eqs. (1) and (8). But Eq. (19) did not apply to a liquid. Indeed, the friction of a particle in a liquid is not caused by collisions of single molecules. As just explained, the effects of these single collisions neutralize each other almost completely. The fluctuations, in which this neutralization is incomplete, may be responsible for all resultant forces on the particle—thus causing the friction as well as the jerky Brownian motion itself—and it may be true[2] that the number of these fluctuations should be proportional to $F^{\frac{1}{2}}$ and thus to the first power of r. Yet, Stokes' law does not follow from this by any simple kinetic considerations. In fact, as will be clear from the hydrodynamical derivation of Stokes' law, the main effects causing the apparent friction are the following: (A) There will be a difference in internal pressure between front and back of the particle, due to a difference in N' at these two spots. This is the main reason for the preference for head-on collisions mentioned in the argument leading to Eq. (1), and has little directly to do with the difference in average relative velocity of molecules and particle at these two spots. (B) Although the friction on the sides of the particle is greatly reduced by the fact that the liquid is taken along by the particle and thus is practically not moving with respect to its surface at all, yet there is a shearing force exerted by the liquid on the particle due to the internal friction of the liquid itself. Thus η enters Eq. (2). Therefore the shearing force depends on the velocity of the particle only indirectly through the *rate of change* of velocity of the *liquid*, with increasing distance from the particle.

From the above it will be clear that the fluctuations in thermal collisions cannot simply be discussed statistically without taking into

[8] Stokes' law, Eq. (2), was not used before the second part of Eq. (15).

TABLE III. Radii and frequencies computed from Stokes' law.*

Liquid	$10^5 \cdot r_{\mathrm{I}1}$ (cm)	$10^5 \cdot r_{\mathrm{I}2}$ (cm)	$10^5 \cdot r_{\mathrm{I}3}$ (cm)	$10^4 \cdot r_{\mathrm{III}} \cdot \rho_0$ (cm^{-2} g)	$10^{10} \cdot r^2 \cdot \rho_0/\tau$ (cm^{-1} sec^{-1} g)	$10^{-5} \cdot f'/\tau$ (cm^{-1} sec^{-1})	$10^{-10} \cdot F_2/r^2$ (cm^{-2} sec^{-1})	$10^{-10} \cdot F_3/r^2$ (cm^{-2} sec^{-1})
Acetone	0.0009	0.0076	0.015	0.048	1.5×10^{8}	4.8×10^{15}	6.5×10^{17}	3.3×10^{17}
Aniline	?	0.061	0.140	0.50	2.0×10^{9}	4.0×10^{16}	6.7×10^{17}	2.9×10^{17}
Benzene	0.053	0.013	0.025	0.090	2.9×10^{8}	7.1×10^{15}	5.5×10^{17}	2.9×10^{17}
Carbon tetrachloride	0.0064	0.0147	0.028	0.187	4.4×10^{8}	5.4×10^{15}	3.6×10^{17}	1.9×10^{17}
Cyclohexanol	?	1.0	2.2†	7.65	(3.1×10^{10})	(5.8×10^{17})	5.8×10^{17}	2.6×10^{17}†
Ethyl acetate	0.0021	0.0094	0.022	0.068	2.0×10^{8}	4.4×10^{15}	4.6×10^{17}	2.1×10^{17}
Ethyl alcohol	0.0084	0.0285	0.045	0.18	5.4×10^{8}	2.2×10^{16}	7.8×10^{17}	4.9×10^{17}
Ethyl ether	0.0047	0.0070	0.012	0.040	1.0×10^{8}	2.6×10^{15}	3.8×10^{17}	2.2×10^{17}
Glycerin	?	14	50†	140	(6.7×10^{11})	(1.4×10^{19})	9.8×10^{17}	2.8×10^{17}†
Mercury	?	0.00178	0.009‡	0.193	7.0×10^{8}	6.6×10^{15}	3.7×10^{18}	7.5×10^{17}††
Methylene bromide	?	0.00097	0.0025†	0.0193	4.7×10^{7}	5.1×10^{14}	5.3×10^{17}	2.0×10^{17}†
Phenol	?	0.18‡	0.33	1.54‡	(5.4×10^{9})	(1.1×10^{17})	6.0×10^{17}‡	3.3×10^{17}
Propyl acetate	?	0.011‡	0.028	0.076‡	2.7×10^{8}	4.9×10^{15}	4.6×10^{17}‡	1.7×10^{17}
Water	0.013	0.015	0.017	0.12	4.5×10^{8}	4.7×10^{16}	3.1×10^{18}	2.7×10^{18}

* The first four columns of figures give r_{I} and r_{III} in units 10^{-5} cm for particles with $\rho_0=1$. For particles smaller than r_{I} or r_{III}, Stokes' law is not valid. (See text.) The last four columns give frequencies in sec^{-1} for particles with $r=10^{-5}$ cm and $\rho_0=1$, calculated as if Stokes' law were always valid. Here $(1/\tau) \approx$ frequency of complete change of direction of particle; $f' \lesssim$ frequency of effective collisions contributing to jerky motion; $F \approx$ frequency of thermal collisions against particle. The subscripts 1, 2, 3 refer to calculation by means of Eq. (17:1), (17:2), or (17:3); for instance, F_3 is obtained by substitution of Eq. (17:3) into Eq. (16). Where Stokes' law gives wrongly $f' > F$ for $r=10^{-5}$ cm, we obviously have a case with $r_{\mathrm{I}} > 10^{-5}$ cm, that is, where for calculation of f' or τ we should not have applied Stokes' law for $r=10^{-5}$ cm. (In the table, this is indicated by putting the values for f' and for τ between brackets.)

† Guessed on basis of the arbitrary assumption that $\rho_{\mathrm{crit}} \approx 50$ atmos.

†† Guessed on basis of the arbitrary assumption $\rho_{\mathrm{crit}} \approx 200$ atmos.

‡ Guessed on basis of the arbitrary assumption $v_0 \approx 1.4\times10^{5}$ cm/sec.

account complicated correlations between the velocities of molecules at different places in the liquid, if we want to explain the friction law, Eqs. (1) and (2). As this is no simple problem, we shall here not enter into a further detailed discussion of the mechanism of these effects. We shall, however, later discuss under what circumstances the mechanisms (A) and (B), in the way in which they are taken into account in the usual hydrodynamical derivation of Stokes' law, *must* fall short of giving an adequate description of the friction.

Before we do so, let us first consider the case of sufficiently large particles, for which Stokes' law, Eq. (2), with satisfactory accuracy gives the magnitude of the friction that is effective in damping the jerky Brownian motion.

We could define the number of effective collisions per second f' by

$$f'm^2\langle V_x^2\rangle_{\mathrm{av}} = 2SkT \tag{20}$$

with $\langle V_x^2\rangle_{\mathrm{av}}$ given by Eq. (18); or, if Stokes' law, Eq. (2), is valid, by

$$f'm^2\langle V_x^2\rangle_{\mathrm{av}} = 12\pi kT\eta r. \tag{21}$$

Comparing this with Eq. (9), we see that f' is thus defined as the number of completely unhindered collisions per second that is just sufficient to give the particle, under the influence of the friction law, Eq. (1) or Eqs. (1) and (2); the correct mean square velocity required by the law of equipartition of energy. We may add that f' is a rather conservative estimate of f, as actually the effective collisions (the ones that in the beginning of Sec. 1 were called "collisions of exceptional vigor") may be *less-than-usually* hindered rather than being completely unhindered. Then, of course, more effective collisions are needed than are given by f'; that is, $f' < f$.

Substituting Eq. (18) into Eq. (21), we find that, for particles big enough for application of Stokes' law,

$$f' = 9\pi r\eta/2\mu. \tag{22}$$

Thus, the larger the viscosity of the liquid, the greater number of thermal collisions should be effective in setting the particle into motion, in order to keep things going, if equipartition of energy is to hold for the particle.

Let t be the time after which the particle in its irregular motion has completely forgotten about its original velocity. This loss of memory is partially due to the friction in the liquid (which leads to $t \approx \tau$). We can also say that the resultant of all impulses during the time t should have a mean square comparable with the average square of the momentum of the particle.[2] Thus

$$\{\sum_{i=1}^{ft} m\mathbf{V}_i\}^2 = (ft)\cdot m^2\langle \mathbf{V}_i^2\rangle_{\mathrm{av}}$$

$$\approx \langle m^2v^2\rangle_{\mathrm{av}} = (2m)(3kT/2)$$

or, by three times Eq. (9),

$$t=m/2S=\tfrac{1}{2}\tau=m/12\pi\eta r. \tag{23}$$

Thus the motion of the particle can be considered as rectilinear only during time intervals much shorter than the relaxation time τ. Comparing the frequency $(1/t)$, with which the particle is changing its direction, with the frequency of effective collisions conservatively estimated in Eq. (22), we see that the latter is larger than $(1/t)$ by a factor

$$f't=3m/8\mu, \tag{24}$$

which is obviously very large.

By Eq. (23), this also means that the relaxation time τ is many orders of magnitude longer than the time between two subsequent effective collisions,[9] contrary to the assumption made in Müller-Pouillet's *Lehrbuch*.[1] Even for a particle near the limit of visibility under a microscope using ordinary light (with $r\approx10^{-5}$ cm), in a liquid with molecular weight $M\approx90$, we find $f'\tau\approx2\times10^{7}\rho_0$, where ρ_0 is the specific gravity of the particle. (Compare also Table III.)

As a consequence of this fact, together with $\langle V^2\rangle_{\text{av}}\div\langle v^2\rangle_{\text{av}}=8\mu/3m\ll1$, (compare Eq. (18) with Eq. (8)), a particle in Brownian motion does not really describe a zigzag motion as is often pictured, but describes a path curved irregularly though not at one point excessively more than at another point. An apparent zigzag motion is obtained only if positions observed at definite time intervals are drawn in a diagram and then connected by straight lines.[10]

While f' as given by Eqs. (21)–(22) is very large compared with $(1/\tau)$, it must inevitably be smaller—often very much smaller—than F. This follows from the above qualitative arguments. For, indeed, the number f of collisions per second that cause the Brownian motion cannot be larger than the *total* number of collisions F given by Eq. (16), and due to the neutralizing effect probably will be much less than F. Our f', defined by Eq. (20), will then obviously be still smaller, so that $f'\ll F$. Also, the square, $m^2V_{iz}^2$, of the x-component of impulse picked up by the particle from a single relatively unhindered collision will, on the average, be not more than $m^2\langle V_x^2\rangle_{\text{av}}$ as given by Eq. (18). Therefore,

$$fm^2\langle V_{iz}^2\rangle_{\text{av}}<Fm^2\langle V_x^2\rangle_{\text{av}}.$$

This relation, which we have already discussed qualitatively in the argument leading to Eq. (19), is now going to give us one condition for the validity of Stokes' law for the friction damping the Brownian motion. For this purpose, we substitute again Eqs. (16) and (18) into its right-hand member, and then use it for finding an upper limit for the right-hand member of Eq. (9). We thus find

$$2SkT<4\pi r^2\cdot\tfrac{1}{4}\langle c\rangle_{\text{av}}\cdot N'\cdot(8\mu kT/3),$$

or, inevitably,

$$S<4\pi r^2N'\mu\langle c\rangle_{\text{av}}/3. \tag{25}$$

Thence, Stokes' law, Eq. (2), can be used in Eqs. (1)–(9) only if $6\pi\eta r<4\pi r^2N'\mu\langle c\rangle_{\text{av}}/3$, that is, a first condition for the validity of Stokes' law for friction in Brownian motion is

$$r>r_1,$$

where

$$r_1=9\eta/2N'\mu\langle c\rangle_{\text{av}}. \tag{26:1}$$

For particles smaller than this limit r_1, Eq. (25) obviously gives $S<4\pi r r_1N'\mu\langle c\rangle_{\text{av}}/3$, or, by Eq. (26:1), $S<6\pi\eta r$, so that for such small particles the friction in Brownian motion will be less than would follow from Stokes' law.

For the Brownian motion of smoke particles suspended in a gas, for which $N'\approx N$, and for which the viscosity η can be expressed in terms of the mean free path l of the molecules roughly by Enskog and Chapman's equation[11] $\eta\approx0.5N\mu\langle c\rangle_{\text{av}}l$, our condition, Eq. (26:1), gives the well-known result[12] that Stokes' law breaks down, if r is not much larger than l. (We actually find $r>9l/4$.) If r is much smaller than l, Stokes' law, Eq. (2), is indeed to be replaced by Epstein's equation[13] $S=4\pi r^2N\mu\langle c\rangle_{\text{av}}/3$, which is in perfect agreement with our Eq. (25) with $N'=N$, and with $<$ re-

[9] Compare also M. von Smoluchowski, *Ann. der Physik* **21**, 756 (1906).

[10] For physiological reasons the motion may perhaps make a similar appearance to the human eye.

[11] For references, see, for instance, E. H. Kennard, *Kinetic theory of gases* (McGraw-Hill, New York, 1938), p. 147, Eq. (126b).

[12] See, for instance, E. H. Kennard, reference 11, p. 310.

[13] P. S. Epstein, *Physical Rev.* **23**, 710 (1924), finds a slightly higher value for S, if he assumes that the colliding molecules are reflected diffusely by the particle. As we did not consider such possibility in our calculation of $\langle V_x^2\rangle_{\text{av}}$ in Eq. (18), the apparent disagreement between such finding and our inequality (25) is not surprising.

placed by $=$, as is to be expected; for $f=F$ and $\langle V_{ix}^2\rangle_{\text{av}}=\langle V_x^2\rangle_{\text{av}}$.

In the case of liquids, we shall take for N' in Eq. (26:1) the value from one of the Eqs. (17:1, 2, 3). Thus, we find for r_{I} in cm

$$r_{\text{I1}}=\frac{3\eta(3\rho_{\text{crit}}-\rho)}{2\langle c\rangle_{\text{av}}\rho\rho_{\text{crit}}}$$

$$=0.60\times10^{-5}\times\frac{\eta(3\rho_{\text{crit}}-\rho)M^{\frac{1}{3}}}{\rho\rho_{\text{crit}}}; \quad (26:\text{I1})$$

$$r_{\text{I2}}=\frac{9\eta}{4\rho v_s}; \quad (26:\text{I2})$$

$$r_{\text{I3}}=\frac{\eta}{12p_{\text{crit}}}\left(\frac{\pi kT}{2\mu}\right)^{\frac{1}{2}}=\frac{1600\times10^{-5}\times\eta}{M^{\frac{1}{3}}p'}. \quad (26:\text{I3})$$

Here, p' is the critical pressure measured in atmospheres; η is the viscosity in cgs units (poises). With $\rho\approx1$ g/cm^3, $v_{\text{sound}}\approx1.4\times10^5$ cm/sec, $p'\approx50$ atmos and $M\approx90$, this gives $r_{\text{I2}}\approx1.6\eta\times10^{-5}$ cm or $r_{\text{I3}}\approx3.4\eta\times10^{-5}$ cm. Here, η will be very different for different liquids. For instance, at room temperature it is 0.001 for methylene bromide and about 15 for glycerin. For glycerin, therefore, Stokes' law has already broken down for Brownian motion of particles of (more than) 10^{-4} cm radius, while for methylene bromide Stokes' law may be valid for the smallest particles visible through the microscope. In general, the applicability of Stokes' law for the friction in Brownian motion will break down sooner (that is, for larger size particles) in liquids of high viscosity than in liquids of low viscosity. Values for r_{I1}, r_{I2}, and r_{I3}, calculated from the data given in Table I, are listed in Table III.

The reasons for our conclusion that Stokes' law breaks down may be reformulated and summarized as follows: By Eq. (16), the number of collisions against a given particle is limited; by Eq. (18), the impulse per collision is limited. If Stokes' law were always applicable to this Brownian motion, the friction would not decrease sufficiently rapidly with decreasing size of the particles (as compared to the decreasing amount of impulses available), so that the impulses would then, for very small particles, not suffice to give them the average speed required for validity of the equipartition law. Since, for Brownian motion, we believe more strongly in this equipartition law than in Stokes' friction law, we must conclude that for such small particles Stokes' law will not be valid.

There are some more conditions limiting the validity of Stokes' law for particles in Brownian motion. While the limitation to particles with $r>r_{\text{I}}$ given above was obtained here rather indirectly and, for liquids,[14] without an attempt to understand the cause of the breakdown of Stokes' law at $r\approx r_{\text{I}}$, the following two limitations will be derived directly by consideration of the hydrodynamical mechanism that explains Stokes' law.

First of all, we note that according to Stokes the pressure in the liquid at the front and at the back of the moving spherical particle would deviate from the average pressure in the liquid by an amount

$$\Delta p=\pm3\eta v/2r.$$

As the total internal pressure due to thermal motion will not be negative, the $(-\Delta p)$ at the back of the particle cannot exceed the average internal pressure $p_{\text{int}}\approx27p_{\text{crit}}$ in the liquid. We thus find that the mechanism leading to Stokes' friction law cannot be effective, unless the condition $(3\eta v/2r)<27p_{\text{crit}}$ is satisfied, or, if we put $v\approx(8kT/\pi m)^{\frac{1}{2}}$ for the particle, we must have

$$r>r_{\text{II}},$$

with

$$r_{\text{II}}=\frac{\eta}{9p_{\text{crit}}}\left(\frac{2kT}{\pi m}\right)^{\frac{1}{2}}. \quad (26:\text{II})$$

As obviously[15]

$$r_{\text{II}}/r_{\text{I3}}=(8/3\pi)(\mu/m)^{\frac{1}{3}}\ll1, \quad (27)$$

we find that this new condition, $r>r_{\text{II}}$, is satisfied automatically as long as $r>r_{\text{I}}$, so that it does not really restrict the validity of Stokes' law any further.

The following condition is more important. It

[14] Note that for a liquid r_{I} need not be as simply related to a mean free path l as for a gas, so that for a liquid there may be reasons for a limitation of the validity of Stokes' law, different from those given by Epstein for a gas (see reference 13).

[15] For further evaluation of r_{II}, one should substitute $m=(4\pi/3)r_{\text{II}}^3\rho_0$ into Eq. (26:II), and then solve for r_{II}. This gives $r_{\text{II}}=(kT\eta^2/54\pi^2\rho_0\rho_{\text{crit}}^2)^{1/5}=0.237\times10^{-5}\eta^{2/5}(p')^{-2/5}\rho_0^{-1/5}$, or, for $p'\approx50$ and $\rho_0\approx1$, $r_{\text{II}}\approx0.05\eta^{2/5}\times10^{-5}$ cm.

requires that the hydrodynamical pattern of currents in the liquid surrounding the particle, in case of validity of Stokes' law, shall be able to establish itself within the time interval which elapses before the velocity of the particle changes its direction again.

As seen above, this time t is about half the relaxation time τ. Even in a time short compared with this t (say in a time $\approx \frac{1}{2}t \approx m/4S$) the pattern should be able to propagate at least over a distance $\approx r$. The speed of this propagation we identify with the velocity of sound in the liquid. This gives the condition $\frac{1}{2}tv_s > r$, so that for Stokes' law to be valid for the friction in Brownian motion, we find ultimately the condition

$$r > r_{\mathrm{III}},$$

with

$$r_{\mathrm{III}} = 18\eta/\rho_0 v_s. \qquad (26:\mathrm{III})$$

Now, since

$$r_{\mathrm{III}}/r_{12} = 8\rho/\rho_0, \qquad (28)$$

we find that (with ρ and ρ_0 usually of the same order of magnitude) the limitation to the validity of Stokes' law for friction in Brownian motion obtained here is slightly stricter than that given by Eq. (26:I). In Table III, values for r_{III} in units 10^{-5} cm are listed for particles with $\rho_0 = 1$ in various liquids. A comparison with the values obtained for r_{I} shows that in all cases r_{III} is indeed a little higher than r_{I}, while $r_{\mathrm{III}}/r_{\mathrm{I}}$ varies much less from one liquid to another than r_{III} and r_{I} vary themselves. This is, of course, due to the common factor η in Eqs. (26:I) and (26:III).

This approximate proportionality of r_{I} and r_{III} might seem to suggest that the mechanism of breakdown of Stokes' law considered in Eq. (26:III) could well be the very mechanism that, by Eq. (26:I), we tacitly assumed to exist in order to save the equipartition law for small particles. However, the fact that r_{III} depends on ρ_0, while r_{I} does not, shows that this is not generally the case. In principle, we might find $r_{\mathrm{I}} > r_{\mathrm{III}}$ for particles of extremely high specific gravity in extremely light liquids (with $\rho_0 > 8\rho$). For such highly exceptional cases, therefore, we have not yet found the mechanism responsible for the first breakdown of Stokes' law for Brownian motion occurring at $r \approx r_{\mathrm{I}}$.[14] However, for the more usual case that $\rho \approx \rho_0$ we still can say that the validity of the equipartition law at least *happens* to be saved by the fact that hydrodynamical current patterns causing Stokes' friction are prevented from being set up around small particles due to the lack of time for the pattern to establish itself within the relaxation time of such particles.

Availability of Radioantimony-Beryllium Neutron Sources

Oak Ridge National Laboratory has developed a relatively inexpensive radioantimony-beryllium neutron source which is now available for distribution to users in the United States. These sources will be allocated through the Isotopes Division, U. S. Atomic Energy Commission.

The sources use a core of 32 grams of antimony metal surrounded by a $\frac{3}{16}$-in. thick beryllium metal cup, the assembly being enclosed in an aluminum jacket. The outside dimensions of this jacket are as follows:

Cylinder containing source	$1\frac{3}{16}$-in. diameter $\times 1\frac{3}{16}$-in. long
Aluminum plate protruding from one end	$\frac{3}{8}$ in.
Hole in plate for handling	$\frac{1}{4}$ in.
Over-all length of assembly	$1\frac{3}{4}$ in.

The entire unit is placed in the pile and the antimony activated to the desired level. Antimony^{122} (half-life 2.8 days) and Antimony^{124} (half-life 60 days, gamma-ray energy 1.72 Mev) are both formed in this irradiation, but the source can stand for two weeks before shipment to allow the Sb^{122} to decay. Saturation activity of the Sb^{124} is approximately 3.5 curies; however, normally an activation time of 120 days will be used which will give an activity of about 2.6 curies. A source activated for 69 days and cooled for 16 days gave an unshielded gamma-radiation reading of about 20 roentgens/hour at one foot. Other readings followed the inverse square very closely. Shorter activation periods may be arranged if a lower activity is satisfactory. After the activity of the Sb^{124} has decayed, it may be reactivated by reinsertion in the pile.

It has been found that the saturated source will emit a total of about 8×10^6 neutrons per second or 9.8×10^4 neutrons/cm^2/second at one inch, if it is assumed that the source behaves as a point source. The neutrons are approximately monoenergetic. Arrangements must be made to provide a suitable storage space for the source; a lead pig with walls four inches thick would probably be sufficient.

On Newton's Third Law and the Conservation of Momentum

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IT is customary in both elementary and advanced treatments of mechanics to derive the conservation laws of linear and angular momentum from Newton's third law of motion.[1] On the other hand, the forces between moving charges do not in general obey Newton's third law. One of the simpler instances of this fact, the problem of two charged particles moving in mutually perpendicular directions, has been discussed in a number of recent papers.[2–4] The usual answer to the difficulty seemingly raised by this failure of Newton's third law has been given by Keller.[2] The conservation laws remain valid when the momentum of the electromagnetic field is taken into account, from which it is concluded that the difficulty is only apparent and not real.

It appears to this writer that the above answer is not wholly to the point. Mechanics is concerned with the motions of material bodies. If these bodies are assumed to consist of Newtonian particles, then Newton's third law explains and elucidates the experimentally observed fact that material bodies conserve mechanical momentum. If the particles are charged, introduction of an electromagnetic field possessing momentum preserves the conservation laws. However, it is scarcely cogent to infer from this demonstration of what amounts to new conservation laws that material bodies, which are known to be a vast conglomeration of moving charges, will conserve mechanical momentum in the absence of external forces. As a matter of fact material bodies do not necessarily conserve momentum in the presence of an electromagnetic field, as is evidenced, for example, by the observable effects of radiation pressure. Conservation of momentum is a fact of experience only because electromagnetic forces are very small, so small as to be usually negligible, and because in addition a body in thermal equilibrium with the radiation surrounding it radiates as much momentum as it receives, on the average. If electromagnetic forces were very much larger, i.e., if the charge on the electron were very much larger, the linear momentum or angular momentum of isolated material bodies might very well show large fluctuations, even when averaged over relatively long time intervals.

This brief discussion exemplifies the obvious point that using Newton's third law to prove the conservation laws of linear and angular momentum involves an assumption: that it makes good sense to regard material bodies as composed of Newtonian particles or, in other words, of mass points obeying Newton's third law. Unfortunately, this assumption is rather difficult to defend. For this reason it is of interest to note that the conservation laws of linear and angular momentum can be justified by an argument which does not depend on special assumptions regarding the nature of the forces between the elementary particles composing material bodies. In particular, it is not necessary specifically to invoke Newton's third law. Naturally this argument also involves a physical assumption. It is sufficient to assume that the internal forces in material bodies do no work in any virtual displacement which is merely an infinitesimal translation or rotation. This assumption is in many respects more in keeping with our present understanding of the nature of matter than is the assumption that material bodies consist of Newtonian particles. The elucidation of the statements of this paragraph is presented in the following section.

Ultimately the proof of the conservation laws of linear and angular momentum must be based on quantum mechanics. In order that a classical proof be meaningful it is necessary that the

[1] We follow Whittaker and explicitly include in Newton's third law the assumption that the forces between two particles are directed along the line joining them. This assumption of central forces was not included in Newton's original formulation of the third law, but as is well known, is required for the usual proofs of the conservation law of angular momentum. Since throughout this paper we treat the conservation laws of linear and angular momentum on an equal footing, we are merely avoiding constant repetition of the awkward phrase "and along the line joining them." E. T. Whittaker, *A treatise on the analytical dynamics of particles and rigid bodies* (Dover Publications, 1944), p. 27.

[2] J. M. Keller, *Am. J. Physics* **10**, 302 (1942).

[3] I. A. Robertson, *Phil. Mag.* **36**, 32 (1945), and **39**, 162 (1947).

[4] W. A. Tripp, *Phil. Mag.* **38**, 149 (1947).

fundamental assumptions on which classical mechanics rests, especially the assumption of particles following definite trajectories, make good sense from the standpoint of quantum mechanics. This paper attempts to demonstrate that while perhaps no defense of the assumptions of classical mechanics can be completely satisfactory, a better defense can be made if it is not specifically required that every one of these particles obey Newton's third law. Although Newton's third law may no longer be obeyed in detail, nonetheless systems of particles continue to behave as if Newton's third law were obeyed, in the sense that the conservation laws of linear and angular momentum remain valid. Moreover, justifying the conservation laws without detailed reference to Newton's third law probably gives more insight into their physical significance than merely inferring them from the subsumed truth of Newton's third law.

It should be remarked that the result, that the invariance of the Lagrangian with respect to translations and rotations implies the conservation laws of linear and angular momentum, has been proved long ago for systems describable by a Lagrangian.[5] (The corresponding invariance of the Hamiltonian guarantees the conservation laws of linear and angular momentum in quantum mechanics.) Basing the proofs of these conservation laws on the existence of a Lagrangian seems, however, less significant physically than the proofs given below. It is by no means obvious that any system of particles has a Lagrangian, or, if there is a Lagrangian, that it depends only on the relative positions and velocities of the particles, and does not depend on any other accessory quantities. To illustrate, the motion of any system of charged particles can be described by a Lagrangian, but the Lagrangian is an explicit function of the electromagnetic field variables, as well as of the particle positions and velocities. The Lagrangian is not invariant with respect to translations or rotations of the particles alone, and consequently mechanical momentum, the momentum of the particles only, is not conserved.

Justification of the Conservation Laws

The usual derivation of the laws governing the rate of change of linear and angular momentum of a system of particles proceeds as follows.[6] The equation of motion of the ith particle of a system is

$$\mathbf{F}_i+\sum_{j\neq i}\mathbf{f}_{ij}=m_i\ddot{\mathbf{r}}_i, \tag{1}$$

where $\mathbf{F}_i$ is the external force acting on the particle and $\mathbf{f}_{ij}$ is the internal force exerted on particle i by particle j. Since $\mathbf{f}_{ij}=-\mathbf{f}_{ji}$, summing Eq. (1) over all particles leads to

$$\sum_i \mathbf{F}_i=\sum_i m_i\ddot{\mathbf{r}}_i. \tag{2}$$

Furthermore, since $\mathbf{f}_{ij}$ is directed along the line between particles i and j, i.e., since $\mathbf{f}_{ij}$ is parallel to $\mathbf{r}_i-\mathbf{r}_j$ it also follows that

$$\sum_i (\mathbf{r}_i\times\mathbf{F}_i)=\sum_i m_i(\mathbf{r}_i\times\ddot{\mathbf{r}}_i). \tag{3}$$

Equations (2) and (3) express the usual rules that the sum of the external forces equals the rate of change of momentum of the system, and the sum of the applied torques equals the rate of change of angular momentum.

Now Newton's third law can either be regarded as an independent postulate regarding the allowed types of forces between particles, or it can be deduced, following Mach,[7] from the definition of mass. In either event, its use to deduce Eqs. (2) and (3) involves the assumptions (a) that any material body, such as a rigid body, is composed of a definite and immutable number of point particles, and (b) that the accelerations of these particles, when no external forces are acting, can always be understood in terms of pairs of forces between pairs of particles, i.e., in terms of two-body forces which obey Newton's third law. When an attempt is made to identify these particles, however, it is apparent that the above assumptions are inconsistent with present understanding of the structure of matter. Thus, if the particles are supposed to be atoms or molecules, it can scarcely be maintained that the internal forces between such particles occur always in pairs which obey Newton's third law. Rather, because atoms and molecules have structures

[5] Whittaker, see reference 1, p. 58.

[6] Cf. e.g., G. Joos, *Theoretical physics* (G. E. Stechert and Co., 1934), pp. 102 ff.

[7] E. Mach, *The science of mechanics* (Open Court Publishing Co., 1942), pp. 266 ff. Cf. also, R. B. Lindsay and H. Margenau, *Foundations of physics* (John Wiley and Sons, Inc., 1936), pp. 91 ff. and H. A. Simon, *Phil. Mag.* **38**, 888 (1947).

which can and do change, as when a molecule is polarized by the electric field of a neighboring atom, the forces between such particles depend not only on their relative distances but also on their relative orientations and on the positions and orientations of neighboring particles as well. Such forces are really many-body forces; the force acting on any one of three contiguous atoms depends on the simultaneous positions, orientations, and polarizations of the other two, and can only with extreme artificiality be regarded as resulting from pairs of equal and opposite forces between the various atom pairs. In any event, the forces between such particles are not necessarily directed along the line connecting them as is evidenced by the simple examples of the electrostatic forces exerted by a dipole on a single charge or on another dipole.

These difficulties are not removed by identifying the particles with electrons and the constituents of atomic nuclei. Electrons obey the laws of quantum mechanics, not classical mechanics, and, as evidenced by interference and diffraction phenomena, are not at all well localized in space in the sense demanded of a Newtonian particle. This makes relatively unimportant the fact that, because of their spins, the forces between two electrons, like the forces between two electric dipoles, are not necessarily directed along the line connecting them. Similar objections can be cited to identifying the particles of Eq. (1) with the protons and neutrons (and presumably also mesons) which compose atomic nuclei.

The foregoing arguments have shown, not that it is impossible to carry through proofs of Eqs. (2) and (3) using Newton's third law, but that such proofs are based on hypotheses which can only with difficulty and by unnatural artifices be reconciled with present knowledge of the structure of matter. Equations (2) and (3) can, however, be established in the following way.

Let the equation of motion of the ith particle be

$$\mathbf{F}_i+\mathbf{f}_i=m_i\ddot{\mathbf{r}}_i, \tag{4}$$

where $\mathbf{f}_i$ is the total internal force exerted on particle i, and may be considered formally defined by Eq. (4) in the form

$$\mathbf{f}_i=m_i\ddot{\mathbf{r}}_i-\mathbf{F}_i=m_i(\ddot{\mathbf{r}}_i-\ddot{\mathbf{s}}_i). \tag{5}$$

In Eq. (5) $\ddot{\mathbf{s}}_i$ is the acceleration the ith particle would have if it were free to move under the influence of $\mathbf{F}_i$ alone. The internal force $\mathbf{f}_i$ is then just proportional to the difference between the acceleration $\ddot{\mathbf{r}}_i$ which the particle i actually receives and the acceleration it would have had if internal constraints, expressed through $\mathbf{f}_i$, were not effective. Equation (4) differs from Eq. (1) in that the internal force $\mathbf{f}_i$ is not necessarily a sum of two-body forces $\mathbf{f}_{ij}$, obeying Newton's third law. Because of this, many of the objections raised above to identifying the particles with atoms or molecules are no longer pertinent. Like electrons, atoms and molecules obey the laws of quantum mechanics and have wave properties, but there is good reason to believe that at ordinary temperatures and pressures the classical approximation of definite, well-localized trajectories is usually valid for atoms and molecules. Even where this approximation is not altogether valid, it is certainly far better for atoms or molecules than for electrons, at ordinary temperatures and pressures. Comparison of the theories of imperfect gases and of electronic conduction illustrates the difference, in terms of approach to physical reality when identified as classical particles, between atoms or molecules and electrons. It is this fact, that it seemingly makes good physical sense to call atoms and molecules particles when the requirement of two-body forces obeying Newton's third law is abandoned, that justifies the argument below based on Eqs. (4) and (5), whereas the use of Eq. (1) to obtain Eqs. (2) and (3) is not justifiable. These qualitative considerations are formulated more precisely at the end of this section.

Granting the validity of Eqs. (4) and (5), the proof of the conservation laws proceeds as follows. Let $\delta\mathbf{r}_i$ be any set of virtual variations of the positions of the particles compatible with the constraints, such that

$$\sum_i \mathbf{f}_i\cdot\delta\mathbf{r}_i=0. \tag{6}$$

Then it follows from Eq. (4) that for this set of virtual variations

$$\sum_i (\mathbf{F}_i-m_i\ddot{\mathbf{r}}_i)\cdot\delta\mathbf{r}_i=0. \tag{7}$$

Equation (7) looks like D'Alembert's principle, but differs from D'Alembert's principle in that

Eq. (7) holds only for those variations for which Eq. (6) is satisfied. With arbitrary types of internal forces Eq. (6), and therefore also Eq. (7), is not, in general, satisfied for all sets of virtual variations compatible with the constraints. For example, in the problem of two particles m_1 and m_2 moving in free space under the influence of their mutual gravitational attractions, the gravitational forces are the internal forces $\mathbf{f}_i$ of Eq. (4), while the external forces $\mathbf{F}_i$ are zero since the system is isolated. For these internal forces Eq. (6) is not satisfied for arbitrary sets of virtual variations of the positions of the two particles, say a set in which $\delta\mathbf{r}_2=0$ but $\delta\mathbf{r}_1$ is along the direction of $\mathbf{f}_1$.

Consider now the two sets of virtual variations:

$$\delta\mathbf{r}_i=\delta\mathbf{u}=\text{a constant.} \tag{8a}$$

$$\delta\mathbf{r}_i=(\delta\boldsymbol{\phi}\times\mathbf{r}_i). \tag{8b}$$

Substitution of Eq. (8a) in Eq. (7) gives

$$\sum_i(\mathbf{F}_i-m_i\ddot{\mathbf{r}}_i)\cdot\delta\mathbf{u}=0. \tag{9}$$

Substituting Eq. (8b) in Eq. (7) gives

$$\begin{aligned}\sum_i(\mathbf{F}_i-m_i\ddot{\mathbf{r}}_i)\cdot(\delta\boldsymbol{\phi}\times\mathbf{r}_i)&=\sum_i[\mathbf{r}_i\times(\mathbf{F}_i-m_i\ddot{\mathbf{r}}_i)]\cdot\delta\boldsymbol{\phi}\\&=\sum_i(\mathbf{r}_i\times\mathbf{F}_i-m_i\mathbf{r}_i\times\ddot{\mathbf{r}}_i)\cdot\delta\boldsymbol{\phi}=0.\end{aligned} \tag{10}$$

Equations (9) and (10) have been derived on the assumption that the variations (8a) and (8b) satisfy Eq. (6). Accepting this assumption for the moment, Eq. (9) implies that if the constraints permit an infinitesimal translation $\delta\mathbf{u}$ of the entire system in any given direction, then the sum of the components of the external forces along that direction equals the rate of change along that direction of the total momentum of the system. Equation (10) means that if the system can be rotated through an infinitesimal angle $\delta\boldsymbol{\phi}$ about an axis through the origin, then the sum of the external torques about that axis equals the rate of change of total angular momentum along that axis. Since an isolated system can, in general, be translated or rotated in an arbitrary fashion, and since any constraints preventing such translations or rotations in a nonisolated system can always be replaced by the imposition of suitable external forces, it follows that Eqs. (9) and (10) are equivalent to Eqs. (2) and (3).

It remains then to consider under what circumstances the variations (8a) and (8b) satisfy Eq. (6). The variations (8a) and (8b) have the property that they leave unchanged the distances, angles and relative velocities between all parts of the system. In fact, even the relative accelerations and higher time derivatives are unchanged. The infinitesimal rotation and translation amount merely to setting the center of mass of the system at a different point in space, with a somewhat different orientation relative to fixed axes in space. The relative motions of the component particles of the system are not changed. There is no more effect on the motion of the system than if the origin had been translated and the coordinate axes rotated. Thus, in order that Eq. (6) be satisfied, it is sufficient that the internal forces do no work in any virtual displacement of the system that leaves all relative distances, angles, velocities, accelerations, etc., invariant. In fact, the equations $\sum_i\mathbf{f}_i\cdot\delta\mathbf{u}=\sum_i\mathbf{f}_i\cdot\delta\boldsymbol{\phi}\times\mathbf{r}_i=0$ are a necessary and sufficient condition for the conservation laws, i.e., these equations are implied by and imply Eqs. (2) and (3).

It is apparently a fact of experience that this condition is satisfied for ordinary material bodies, or, in other words, the internal forces of material bodies do no work in virtual displacements which merely translate and reorientate the body in space. Certainly there is no evidence either from mechanics or thermodynamics that a translation or rotation of an isolated macroscopic body can change its internal energy; presumably, if the internal energy is unchanged in such a translation or rotation the internal forces have done no work. If the internal energy of the body depends only on the relative distances, angles, velocities, etc., of its component parts, there is obviously no change in internal energy during translations or rotations. The mere requirement that the concept *internal energy of a body* have meaning is almost sufficient to guarantee that the internal energy depends only on relative distances, angles, velocities, etc. Nonetheless, the statement that the internal forces do no work, or the internal energy does not change during translations or rotations remains an assertion justifiable only on grounds of experience. Actually the particles of a body are

not at rest but are vibrating about positions of equilibrium, so that the internal forces may be doing work at any given instant, and there is a continual exchange of energy between the body and the vacuum. However, on the average, the internal energy does remain constant or nearly constant as long as the average relative distances, velocities, etc., of the particles are unchanged.

The demonstration of the conservation laws therefore involves basically the assumption that space is isotropic; if space is isotropic, the internal energy of an isolated system cannot depend on position or orientation relative to an external set of fixed axes, and the conservation laws then follow. The proofs of the conservation laws which rest on the invariance of the Lagrangian or Hamiltonian with respect to translations and rotations[8] also involve basically the assumption that space is isotropic. There is no need to justify the assumption of isotropy for truly empty space, which is isotropic almost by definition. Mechanical systems are embedded, however, not in empty space but in a space containing electromagnetic fields, neutrino fields, etc. Only if the "system" includes all such fields can it be said to be embedded in an empty space whose isotropy implies generalized momentum conservation laws. If the fields are omitted from the system, i.e., if the system is thought to be defined completely by specifying the positions and velocities of its component mass-points, the mechanical momentum of an isolated system is conserved only under specific though widely occurring circumstances. The necessary and sufficient condition for such conservation, or alternatively, the necessary and sufficient condition that space appear isotropic to the mechanical system, is that the forces acting on the collection of mass-points which form the mechanical system do no net work in translations or rotations.[8]

[8] Neither this statement nor any other part of this paper is really new. The essence of the derivation of the conservation laws which has been given here is contained in Whittaker (reference 1). However, the details of the argument, whereby the conservation laws are deduced from an assumption of isotropy without direct reference to a Lagrangian, are not given explicitly in any source known to the writer, (e.g., in such standard works as Whittaker, Appell, the *Handbuch der physik*, or in G. D. Birkhoff, *Dynamical systems*, American Mathematical Society Colloquium Publications, v. 9, 1927). More importantly, the physical significance of the work done by the forces in translations and rotations, and the relationship of this work to the assumption of isotropy, is nowhere sufficiently stressed.

The foregoing paragraphs lead to the following summarizing conclusion. It can be proved that the conservation laws of linear and angular momentum are valid provided the internal forces of a system do no work during translations or rotations of the system. For an isolated body the assertion that the internal energy of the body does not change for translations or rotations implies that the internal forces do no work in translations or rotations. It is not inconceivable that the internal energy may change and the internal forces may therefore do work, but it appears from experience that the energy changes are negligible for ordinary macroscopic bodies (recalling that the internal forces always are defined by Eq. (5)). It follows then that such a body will behave as if the forces between its component parts obeyed Newton's third law.[9] Similarly, in a collision between two macroscopic bodies, it can be assumed that the internal energy does not change during translations or rotations of the system as a whole, so that the bodies will interact as if the forces obeyed Newton's third law, i.e., linear and angular momentum will be conserved during the collision.

To complete the argument, some remarks concerning the validity of Eq. (4) should perhaps be made. It is stressed that the problem of the applicability of Eqs. (4) or (1) is wholly divorced from the problem of the validity of Newton's third law. Whether atoms, molecules, or electrons are regarded as the fundamental classical particles, Newton's third law remains invalid. For Eq. (4) to be applicable to a system of quantum-mechanical particles, (a) the particles must be represented by wave packets which spread sufficiently slowly that a trajectory can be inferred, and (b) the wave packets must not overlap, especially when the system is composed of identical particles. These requirements are generally necessary. It can be shown that such packets can be constructed for the molecules of a gas

[9] It should be noted that it is not possible to justify conservation of linear and angular momentum of a rigid body simply on the grounds that the isolated body at rest remains at rest, implying that the internal forces must form a system whose sum and resultant moment are zero. Whenever the body translates nonuniformly or rotates, strains are set up which cause the internal forces to differ from those obtaining when the body is at rest.

provided that

$$\frac{Nh^3}{(2\pi mkT)^{\frac{3}{2}}} \ll 1. \tag{11}$$

The inequality (11) is the well-known condition for the validity of classical statistics;[10] it is known to be satisfied for all gases at temperatures above 1°K, but is not satisfied for assemblies of free electrons at temperatures below 2000°K.

On the other hand, such packets cannot always be constructed for atoms or molecules. In diamond, for example, the failure of the law of Dulong and Petit shows that the carbon atom is almost never in states of high quantum number. Moreover, even in gases, it is still necessary to demonstrate that the internal coordinates of the molecule can be neglected, so that the molecule can be regarded as a quantum-mechanical particle and characterized by a single coordinate, that of its center of mass. This means that the motion of the center of mass must be separable from the internal motions, which is just another way of saying that the Hamiltonian describing the isolated molecule must be invariant to translations. (It can, of course, be invariant on the average only, because of interaction with the electromagnetic field of the vacuum.)

In fine, it is psychologically unsatisfying, from the standpoint of classical mechanics, simply to assume *ab initio* that the acceleration of the center of mass of an isolated macroscopic system is proportional to the external forces only. The classical-mechanical proof of the conservation laws divides the system into subaggregates, each of which presumably conserves momentum when isolated. The quantum-mechanical justification of this assumption, if it can be given at all, involves an appeal to the invariance of the Hamiltonian or total energy with respect to translations. A classical proof of the conservation laws based on a similar invariance of the energy of the entire macroscopic system is more defensible and probably more significant than a proof based wholly on the assumption that the subaggregates obey Newton's third law.

I should like to thank Dr. Sidney Benson for his comments.

[10] R. H. Fowler, *Statistical mechanics* (Cambridge, 1936), p. 55.

Network Representation of Input and Output Admittances of Amplifiers

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INPUT and output admittances are among the most important characteristics of amplifiers. They are defined as the ratios of the input (output) current to the input (output) voltage and are computed on the basis of the equivalent circuit. Nichols[1] and Miller[2] derived the expression for the input admittance of a linear triode in 1919; Chaffee[3] in 1929 gave the first general derivation and a systematic analysis of both input and output admittances.

This analysis, because of the complexity of the expressions, is usually obtained by considering separately the real and the imaginary components and introducing such simplifying or limiting assumptions as the case may suggest. It is the purpose of this paper to show that the input and output admittances may be represented as the admittances of linear passive networks, obtained from the equivalent circuit with certain additions of parallel or series branches. The consideration of these networks offers direct and physical interpretation of the impedance, showing the importance of the components and allowing the use of graphic phasor analysis. Throughout the following the assumptions of linearity and of negative grid bias will be made.

[1] H. W. Nichols, "The audion as a circuit element," *Physical Rev.* **13**, 404–414 (1919).
[2] J. M. Miller, "Dependence of input impedance of a three-electrode vacuum-tube upon the load in the plate circuit," *Bur. Standards Bulletin*, N. 351 (1919).
[3] E. L. Chaffee, "Equivalent circuits of an electron triode and the equivalent input and output admittances," *Proc. IRE* **17**, 1633–1648 (1929).

Theory

The correspondence between the input variables (V_1, I_1) and the output variables (V_2, I_2) in a linear quadripole may be expressed by means of a bilinear transformation of one of the following types:

$$\begin{cases} V_1 = Z_{11}I_1 + Z_{12}I_2 \\ I_1 = Z_{21}I_1 + Z_{22}I_2 \end{cases} \tag{1}$$

$$\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{cases} \tag{2}$$

$$\begin{cases} V_1 = A_{11}V_2 + A_{12}I_2 \\ I_1 = A_{21}V_2 + A_{22}I_2. \end{cases} \tag{3}$$

If the polarities are defined as in Fig. 1, the coefficients of Eqs. (1), (2) and (3) can be computed or measured from the quadripole on the basis of relations of such type as

$$Z_{11} = (V_1/I_1)_{I_2=0}, \quad Y_{11} = (I_1/V_1)_{V_2=0}, \quad A_{11} = (V_1/V_2)_{I_2=0}, \quad \cdots.$$

For instance $Z_{11}(Y_{11})$ is the input impedance (admittance) at open (short) output circuit, $-Z_{22}(-Y_{22})$ is the output impedance (admittance) at open (short) input circuit.

The systems (1), (2), (3) are unequivocally determined by the matrices of their coefficients, which we shall indicate by $\|Z\|$, $\|Y\|$, $\|A\|$ respectively. Among the elements of these matrices the following relations exist:

$$\|Z\| = \frac{1}{|Y|}\begin{Vmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{Vmatrix} = \frac{1}{A_{21}}\begin{Vmatrix} A_{11} & -|A| \\ 1 & -A_{22} \end{Vmatrix} \tag{4}$$

$$\|Y\| = \frac{1}{|Z|}\begin{Vmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{Vmatrix} = \frac{1}{A_{12}}\begin{Vmatrix} A_{22} & -|A| \\ 1 & -A_{11} \end{Vmatrix} \tag{5}$$

$$\|A\| = \frac{1}{Z_{21}}\begin{Vmatrix} Z_{11} & -|Z| \\ 1 & -Z_{22} \end{Vmatrix} = \frac{1}{Y_{21}}\begin{Vmatrix} -Y_{22} & 1 \\ -|Y| & Y_{11} \end{Vmatrix} \tag{6}$$

where $|Z|$, $|Y|$, $|A|$ are the determinants of $\|Z\|$, $\|Y\|$, $\|A\|$ respectively. The circuit analysis of a linear quadripole may, therefore, be handled by means of the matrix theory, which presents some organizational advantages. Brown and Bennett[4] have shown recently the application of the theory to the solution of vacuum tube problems. A linear amplifier may be represented by its equivalent circuit which constitutes an active quadripole. To find the matrix of the latter, one can apply Kirchhoff's equations or consider the quadripole as a combination of m simpler networks. In the latter case, assuming for example $m=2$, one has $\|A\| = \|A_1\| + \|A_2\|$ if the combination is in cascade, $\|Z\| = \|Z_1\| + \|Z_2\|$ if it is in series, $\|Y\| = \|Y_1\| + \|Y_2\|$ if it is in parallel. For instance the equivalent circuit of a grounded-cathode amplifier may be considered as the parallel combination of the basic passive quadripole and of the active quadripole which represents the internal constant current generator shown in Fig. 2. Its admittance matrix is written therefore

$$\begin{Vmatrix} Y_1+Y_2 & -Y_2 \\ Y_2 & -Y_2-Y_3 \end{Vmatrix} + \begin{Vmatrix} 0 & 0 \\ -\mu Y_3 & 0 \end{Vmatrix} = \begin{Vmatrix} Y_1+Y_2 & -Y_2 \\ Y_2-\mu Y_3 & -Y_2-Y_3 \end{Vmatrix}.$$

The input admittance of a quadripole closed on a load of admittance Y_L (impedance Z_L) may be obtained from Eq. (3) as

$$Y_{in} = \frac{I_1}{V_1} = \frac{A_{21}V_2 + A_{22}I_2}{A_{11}V_2 + A_{12}I_2} = \frac{A_{21} + A_{22}Y_L}{A_{11} + A_{12}Y_L}.$$

Using Eq. (6) there results

$$Y_{in} = \frac{-|Y| + Y_{11}Y_L}{-Y_{22} + Y_L} = \frac{Z_L - Z_{22}}{Z_{11}Z_L - |Z|} = \frac{1}{Z_{in}}. \tag{7}$$

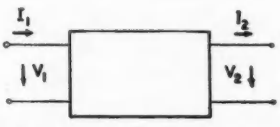


FIG. 1. Definition of polarities in a quadripole.

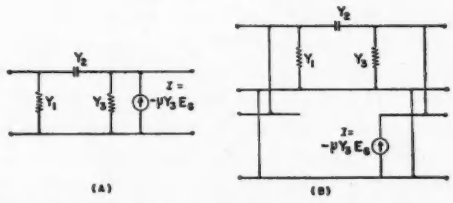


FIG. 2. Equivalent circuit of grounded-cathode amplifier as parallel combination of an active and a passive quadripole.

[4] J. S. Brown and F. D. Bennett, "The application of matrices to vacuum tube circuits," *Proc. IRE* **36**, 844–852 (1948).

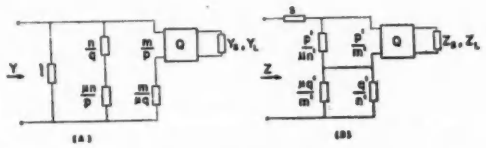


FIG. 3. Network representation of Eqs. (11) and (12).

Similarly the output admittance of a quadripole with a source admittance Y_S (impedance Z_S) is

$$Y_{out}=\frac{-|Y|-Y_{22}Y_S}{Y_{11}+Y_S}=\frac{Z_S+Z_{11}}{-Z_{22}Z_S-|Z|}=\frac{1}{Z_{out}}. \quad (8)$$

Division of numerator by denominator in Eqs. (7) and (8) gives expressions of type

$$Y=l+\frac{m+\mu n}{p+\mu q} \quad (9)$$

$$Z=s+\frac{p'+\mu q'}{m'+\mu n'} \quad (10)$$

where $l+m/p$ is the admittance ($s+p'/m'$ being the impedance) of the corresponding passive quadripole obtained when $\mu=0$.

The problem of finding a 2-terminal network with the given admittance (9) or impedance (10) may be solved in several ways. For instance we may write Eq. (9) in the form

$$Y=l+\frac{1}{p/m+\mu(q/m)}+\frac{1}{q/n+p/\mu n}, \quad (11)$$

or Eq. (10) in the form

$$Z=s+\frac{1}{m'/p'+\mu(n'/p')}+\frac{1}{n'/q'+(m'/\mu q')}, \quad (12)$$

which are represented by the networks of Fig. 3(A, B); the circuit elements are represented as admittances in Fig. 3(A) and as impedances in Fig. 3(B). In this representation q together with the parallel branch l in (a) or with the series branch s in (b) constitutes the basic passive quadripole, to which the representation is reduced when $\mu=0$. Simpler networks may be obtained in particular cases, when q or $p=0$ in Eq. (10), n or $m=0$ in Eq. (11), as will be shown in the following examples.

In Table I are condensed the results obtained for the fundamental cases of the grounded-cathode, grounded-plate, grounded-grid amplifiers. The matrices of their respective equivalent circuits are:

For the grounded-cathode amplifier:

$$\|Y\|=\begin{Vmatrix} Y_1+Y_2 & -Y_2 \\ Y_2-\mu Y_3 & -Y_2-Y_3 \end{Vmatrix}.$$

For the grounded-plate amplifier:

$$\|Y\|=\begin{Vmatrix} Y_1+Y_2 & -Y_2 \\ Y_2+\mu Y_3 & -Y_2-Y_3-\mu Y_3 \end{Vmatrix}.$$

For the grounded-grid amplifier:

$$\|Z\|=\begin{Vmatrix} Z_1+Z_2 & -Z_2 \\ Z_2+\mu Z_2 & -Z_2-Z_3-\mu Z_2 \end{Vmatrix}.$$

In the latter case impedances have been used instead of admittances for convenience of writing formulas.

With reference to the notations of Table I and to the formal representations (9) and (10), the following results are obtained:

For the grounded-cathode amplifier:

$$Y_{in}=Y_1+\frac{Y_2(Y_3+Y_L)+\mu Y_2Y_3}{Y_2+Y_3+Y_L}=l+\frac{m+\mu n}{p}$$

$$Y_{out}=Y_3+\frac{Y_2(Y_1+Y_S)+\mu Y_2Y_3}{Y_1+Y_2+Y_S}=l+\frac{m+\mu n}{p}.$$

For the grounded-plate amplifier:

$$Y_{in}=Y_1+\frac{Y_2(Y_3+Y_L)}{Y_2+Y_3+Y_L+\mu Y_3}=l+\frac{m}{p+\mu g}$$

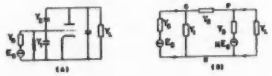
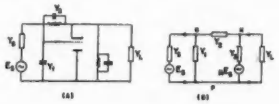
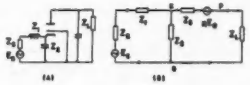
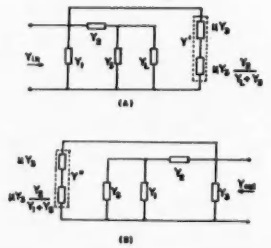
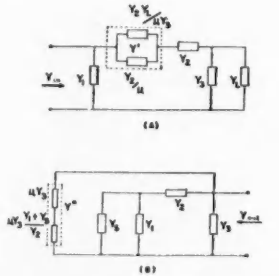
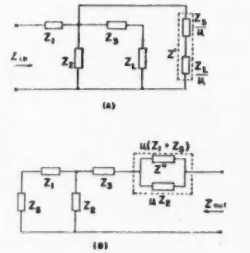
$$Y_{out}=Y_3+\frac{Y_2(Y_1+Y_S)+\mu Y_3(Y_1+Y_S)}{Y_1+Y_2+Y_S}=l+\frac{m+\mu n}{p}.$$

For the grounded-grid amplifier:

$$Z_{in}=Z_1+\frac{Z_2(Z_3+Z_L)}{Z_2+Z_3+Z_L+\mu Z_2}=s+\frac{p'}{m'+\mu n'}$$

$$Z_{out}=Z_3+\frac{Z_2(Z_1+Z_S)+\mu Z_2(Z_1+Z_S)}{Z_1+Z_2+Z_S}=s+\frac{p'+\mu q'}{m'}.$$

TABLE I. Network representation of Y_{in} (or Z_{in}), Y_{out} (or Z_{out}) for linear amplifiers.

Type of amplifier	Grounded-cathode amplifier	Grounded-plate amplifier	Grounded-grid amplifier
Actual and equivalent circuit			
Input admittance Y_{in} or impedance Z_{in}	$Y_{in}=Y_1+\dfrac{Y_2(Y_3+Y_L)}{Y_2+Y_3+Y_L}+\mu Y_3\dfrac{Y_2}{Y_2+Y_3+Y_L}$	$Y_{in}=Y_1+\dfrac{1}{\dfrac{1}{Y_2}+\dfrac{1}{Y_3+Y_L}+\dfrac{\mu Y_3}{Y_2(Y_3+Y_L)}}$	$Z_{in}=Z_1+\dfrac{1}{\dfrac{1}{Z_3+Z_L}+\dfrac{1}{Z_2}+\dfrac{\mu}{Z_3+Z_L}}$
Output admittance Y_{out} or impedance Z_{out}	$Y_{out}=Y_3+\dfrac{Y_2(Y_1+Y_S)}{Y_1+Y_2+Y_S}+\mu Y_3\dfrac{Y_2}{Y_1+Y_2+Y_S}$	$Y_{out}=Y_3+\dfrac{1}{\dfrac{1}{Y_2}+\dfrac{1}{Y_1+Y_S}}+\dfrac{\mu Y_3}{Y_2\left(\dfrac{1}{Y_2}+\dfrac{1}{Y_1+Y_S}\right)}$	$Z_{out}=Z_3+\dfrac{1}{\dfrac{1}{Z_2}+\dfrac{1}{Z_1+Z_S}}+\dfrac{1}{\dfrac{1}{\mu Z_2}+\dfrac{1}{\mu(Z_1+Z_S)}}$
Network representation of $Y_{in}(Z_{in})$ $Y_{out}(Z_{out})$			
Admittances (impedances) of added branches	$Y'=\mu Y_3\dfrac{Y_2}{Y_2+Y_3+Y_L}$ $Y''=\mu Y_3\dfrac{Y_2}{Y_1+Y_2+Y_S}$	$Y'=\dfrac{Y_2}{\mu}\left(1+\dfrac{Y_L}{Y_3}\right)$ $Y''=\dfrac{\mu}{Y_2}\dfrac{Y_3}{\dfrac{1}{Y_2}+\dfrac{1}{Y_1+Y_S}}$	$Z'=\dfrac{Z_3+Z_L}{\mu}$ $Z''=\mu Z_2\dfrac{Z_1+Z_S}{Z_1+Z_2+Z_S}$

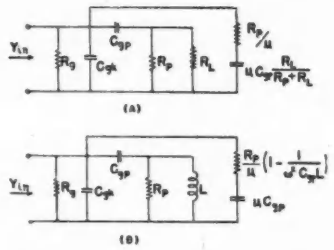


FIG. 4. Network representation of the input admittance of a linear grounded-cathode amplifier (A) with resistive load, (B) with inductive load.

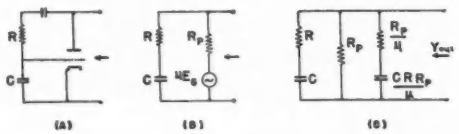


FIG. 5. Network representation of the reactance tube and its output admittance.

The reader may verify directly the derivation of the corresponding network representations as indicated in Table I.

When the amplifier circuit is complicated with feedback branches the simplifications in Eqs. (9) and (10) may not take place and the network representation is of the general type of Fig. 3.

Examples

The above method of representation may be applied in many cases with advantage. For instance, in the classical case of the grounded-cathode amplifier with resistive load, the network representation assumes the form of Fig. 4(A), where $Y_1=(1/R_g)+j\omega C_{gk}$, $Y_2=j\omega C_{gp}$, $Y_3=(1/R_p)$, $Y_L=(1/R_L)$ and

$$\frac{1}{Y'}=\frac{R_p}{\mu}+\frac{1}{j\omega\mu C_{gp}}\left(1+\frac{R_p}{R_L}\right).$$

This shows how the input capacitance approaches a maximum value of $C_{gk}+(\mu+1)C_{gp}$ when the load impedance is very high.[5]

When the load is inductive, the above network assumes the form of Fig. 4(B), where

$$\frac{1}{Y'}=\frac{R_p}{\mu}\left(1-\frac{1}{\omega^2C_{gp}L}\right)+\frac{1}{j\omega\mu C_{gp}}.$$

A negative resistance is then obtained, provided $\omega^2C_{gp}L<1$.

Another interesting example is offered by the reactance tube, Fig. 5. Neglecting the capacitance C_{gp}, we have $Y_1=j\omega C$, $Y_2=1/R$, $Y_3=1/R_p$, $Y_S=0$; consequently

$$\frac{1}{Y''}=\frac{R_p}{\mu}+j\omega\frac{CRR_p}{\mu}.$$

The output admittance of the reactance tube is then represented by the network of Fig. 5(C), which gives complete information on the actual operation of the tube, often approximated by reducing the whole admittance to $j\omega CRR_p/\mu$.

Other examples could be worked out easily; for instance one can verify that the input admittance of a cathode follower (grounded-plate amplifier) presents a negative conductance when the load is capacitive.[6] In our equivalent network the admittance Y' of the added branch is

$$Y'=\frac{Y_2}{\mu}\left(1+\frac{Y_L}{Y_3}\right)=\left(\frac{1}{\mu R_g}+\frac{j\omega C_{gk}}{\mu}\right)(1+R_pY_L)$$

$$=\frac{1}{\mu R_g}+j\omega\frac{C_{gk}}{\mu}+Y_L\left[\frac{R_p}{\mu R_g}+j\omega\frac{C_{gk}R_p}{\mu}\right].$$

So the real component of $Y'-(1/\mu R_g)$ is positive or negative accordingly if the load is inductive ($Y_L=1/j\omega L$) or capacitive ($Y_L=j\omega C$).

[5] F. E. Terman, *Radio engineering* (McGraw-Hill, New York, 1947), p. 365.

[6] K. Schlesinger, "Cathode follower," *Proc. IRE* **33**, 843–855 (1945). H. Reich, "Input admittance of cathode follower amplifiers," *Proc. IRE* **35**, 573–576 (1947).

Anybody can make history; only a great man can write it.—OSCAR WILDE.

Albert A. Michelson at Case

R. S. SHANKLAND
Case Institute of Technology, Cleveland, Ohio

ALBERT ABRAHAM MICHELSON (Fig. 1) was the first professor of physics at Case Institute of Technology (then Case School of Applied Science), holding that position from 1881 until 1889. During these years he performed some of his most important researches in optics. Michelson was born in Strelno, Germany, now Strezelno, Poland, on December 19, 1852. The family moved to Virginia City, Nevada when he was a small boy and later to San Francisco where Albert attended high school. It was his father's wish that Albert become a naval officer, so the young man took the examination for the U. S. Naval Academy which he passed satisfactorily, only to find that all the Congressional appointments had already been filled. Nothing daunted, young Michelson traveled from California to Annapolis to apply personally to the Superintendent of the Academy for admission. He must have made a remarkable impression because an interview was arranged permitting Michelson to present his case in person to President Grant. Here too he met disappointment as Grant had already made the ten presidential appointments at his disposal. However, Michelson so impressed the President with his serious determination and abilities that Grant made an exception and appointed him as an extra man for the Naval Academy that year.

Michelson was graduated from the Naval Academy in the class of 1873 and after spending the usual two years at sea as a passed midshipman returned to the Academy as instructor in physics and chemistry from 1875 until 1879. It was during these years that he was told to "explain the velocity of light to the midshipmen." To accomplish this "laboratory demonstration," Michelson set up in November 1877,[1] simple equipment following Foucault's rotating mirror method[2] of 1862 and obtained results accurate to an unexpected degree. This led him to refine the method using a longer light path along the north sea-wall at the Naval Academy, and in 1879 he could report the value of 299,910±50 kilometers per second for the velocity of light in vacuum.[3] At the same time Simon Newcomb, then superintendent of the Nautical Almanac Office, was engaged in an elaborate determination of the velocity of light over paths extending from Fort Meyer in Virginia to the Washington Monument and the old Naval Observatory. He arranged for Michelson's transfer from the Naval Academy to the Nautical Almanac Office and for one year, 1879–1880, they collaborated in these experiments. Michelson's value for the velocity obtained while working for Newcomb was 299,860±30 km/sec. This result was reported in 1891[4] in the complete summary of Newcomb's researches on the velocity of light.

From 1880 until 1882 Michelson studied and carried on researches in Europe, principally at the Universities of Heidelberg and Berlin and at Le

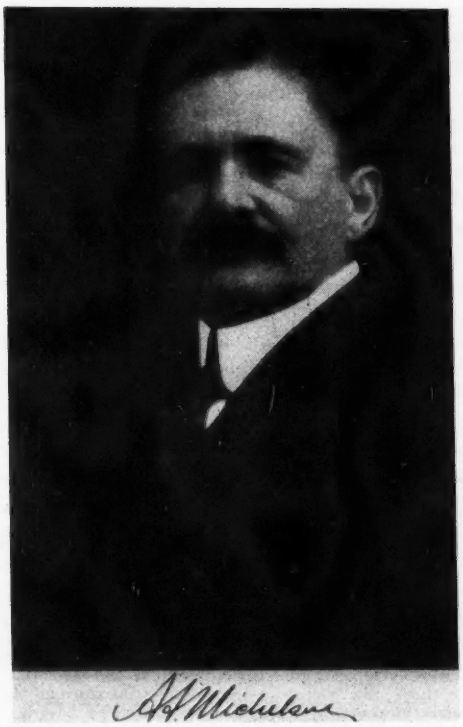


FIG. 1. ALBERT A. MICHELSON in 1881.

[1] A. A. Michelson, St. Louis Meeting of A.A.A.S., 1878; *Am. J. Sci.* **15**, 394 (1878); **17**, 324 (1879); **18**, 390 (1879).
[2] L. Foucault, *Compt. Rend.* **55**, 501 (1862); **55**, 792 (1862).
[3] A. A. Michelson, *Astr. Papers, U. S. Nautical Almanac Office*, **I**, 141 (1882).
[4] Simon Newcomb, *Astr. Papers, U. S. Nautical Almanac Office*, **II**, 201 (1891).

Collège de France in Paris. He was enabled to resign his commission in the Navy and go to Europe because he was assured by the trustees of the newly established Case School of Applied Science that he would be made the first professor of physics at that institution. In fact he was supplied certain funds for travel and for the purchase of the physical laboratory equipment needed at Case. During this period abroad Michelson invented his famous interferometer. The first model of this instrument was built in Berlin in 1881 by Schmidt and Haensch with funds provided by Alexander Graham Bell. This instrument was demonstrated by Michelson in Paris before Cornu and other leading French scientists. Cornu was very outspoken in his skepticism of Michelson's explanation of the performance of the instrument, asserting that the interference fringes were due to some phenomenon analogous to a Lloyd mirror effect in the half-silvered plate. However, Michelson demonstrated that the fringes disappeared when a piece of glass was placed in one of the optical paths beyond the half-silvered plate, and Cornu was convinced.

While in Europe in 1881, Michelson received his official appointment as professor of physics at Case School of Applied Science and was granted leave of absence to continue his researches in Europe during the academic year 1881–1882. During this period he made the first unsuccessful attempt to detect the motion of the earth through space by means of his interferometer. Several factors contributed to this result. Michelson had been led to try the experiment upon reading a suggestion by Clerk Maxwell[5] that the motion of the earth might be detected by measures of the velocity of light if sufficiently refined techniques were available. Michelson's new interferometer provided just the experimental method needed. However, Michelson seems to have expected a first-order effect due to the earth's speed relative to that of light, and so used only a single transit and return of the light beams along the interferometer arms which were 120 cm in length. Two trials of the experiment were made: the first at the Physikalisches Institut of the University of Berlin, where vibration due to street traffic interfered; and the second in a hollow space in the brick pier below the large telescope of the Astrophysicalisches Observatorium at Potsdam, where the experiment gave a negative result.[6]

In September of 1882 Michelson assumed his duties at Case in Cleveland and here began a notable series of researches which firmly established his position in physics. During this first college year of 1882–1883 he made a precise determination of the velocity of light, the accuracy of which was not improved upon until his own experiments were completed in California in 1927. This determination was made at the suggestion of Simon Newcomb[7] who obtained financial support for the work from the Bache Fund of the National Academy of Sciences. An optical path nearly ½ mile long was surveyed along the north side of the tracks of the New York, Chicago and St. Louis Railroad (Nickel Plate) at the rear of the Case campus. This survey was made by the Case Professor of Civil Engineering, John Eisenmann, along the north rail of the track and offset to two concrete piers upon which the optical equipment was mounted. The base line was measured as 2049.5253 feet.[8] The arc light, dynamo and gas engine used to obtain a powerful light source were supplied by the inventor Charles F. Brush, of Cleveland. The rotational speed of the mirror was determined with the aid of an electrically driven Koenig tuning fork of nominal frequency 128 cycles/sec. Michelson made 23 complete determinations of the velocity of light in the period from October 12 until November 14, 1882 and gave as the mean value 299,853±60 km/sec for the velocity of light reduced to vacuum.[9] This result was accepted as the best available measure for forty-five years.

[5] James Clerk Maxwell; article on Ether in Vol. VIII of 9th Ed. of the *Encyclopedia Britannica:* " . . . If it were possible to determine the velocity of light by observing the time it takes to travel between one station and another on the earth's surface, we might, by comparing the observed velocities in opposite directions determine the velocity of the aether with respect to these terrestrial stations. All methods, however, by which it is practicable to determine the velocity of light from terrestrial experiments depend on the measurement of the time required for the double journey from one station to the other and back again, and the increase of this time on account of the relative velocity of the aether equal to that of the earth in its orbit would be only about one hundred millionth part of the whole time of transmission, and would therefore be quite invisible."

[6] A. A. Michelson, *Am. J. Sci.* **22**, 120 (1881); *Phil. Mag.* **13**, 236 (1882); *Am. J. Sci.* **23**, 395 (1882); *J. de Physique* **1**, 183 (1882).

[7] Simon Newcomb, *Astr. Papers, U. S. Nautical Almanac Office*, **II**, 235 (1891).

[8] Report of John Eisenmann to A.A.M., Oct. 14, 1882.

[9] A. A. Michelson, *Astr. Papers, U. S. Nautical Almanac Office*, **II**, 243 (1891); *Am. J. Sci.* **31**, 62 (1886).

During Michelson's second year at Case, 1883–1884, he measured the velocity of light in distilled water. The optical path between mirrors in this experiment was reduced to 17.63 feet, of which 10.03 feet were in the tube containing the distilled water. The result expressed as the ratio of his determination of the velocity in vacuum to the velocity in distilled water was found to be 1.33.[10] At that time this result was received as strong evidence for the wave theory of light. Foucault and also Fizeau and Breguet, at the suggestion of Arago, had previously shown qualitatively that light traveled more slowly in water than in air,[11] but no quantitative measurements were available until those of Michelson.

Michelson next filled the tube with carbon disulphide and determined the velocity of white light as well as the velocity of red and blue light in this medium. Here he obtained results which he had in no way anticipated. The ratio of the velocity in vacuum to that in CS_2 he found to be 1.758 ± 0.02, whereas the index of refraction was known (Verdet) to be 1.64. He also found that the ratio of the velocities of red and blue light was 1.014, the first case he had found of dispersion. In his report to Simon Newcomb[12] he wrote ". . . it is with great diffidence that I state that the velocity of light in carbon disulphide is to that in air as 1.00 to 1.76 . . ."

These results, however, received an immediate explanation in terms of Lord Rayleigh's[13] theory which predicted the difference between the group velocity, which Michelson's method measured, and the phase velocity given by measurements of the index of refraction. These results attracted notice throughout the world of physics and enhanced Michelson's reputation as a leader in experimental optics.[14]

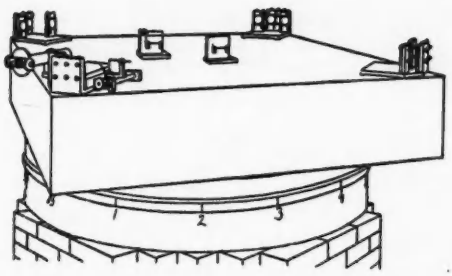


FIG. 2. Michelson-Morley interferometer used in Cleveland in 1887. The illustration is taken from *The American Journal of Science*, Series 3, **34**, 337 (1887). This journal is frequently known as "Silliman's Journal," after its founder, Benjamin Silliman, Professor of Chemistry at Yale University.

In the year 1885 began the famous collaboration between Michelson and Professor Edward W. Morley of Western Reserve University which continued until Michelson left Case four years later. Their first experiment was to measure the velocity of light in moving water to test the theory of Fresnel which predicted that the velocity would be altered by a fraction $(n^2-1)/n^2$ of the velocity of translation of the medium, where n is the index of refraction. The expected fraction for water was 0.438. For this experiment an adaptation of the Michelson interferometer was employed and the measurement recorded the difference between the velocity in the column of water when in motion and at rest; the experimental technique being far superior to that employed by Fizeau[15] in 1851. Their experimental result gave a factor of 0.434 ± 0.02 in remarkable agreement with theory.[16]

Following this work Michelson and Morley performed the interferometer experiment which bears their name and upon which their fame chiefly rests. H. A. Lorentz[17] had shown that the effect to be expected in this experiment should be a second-order one, which partly explained why Michelson's Berlin experiments designed to measure the larger first-order effect were failures. Michelson and Morley accordingly designed and built a large multiple path interferometer, with the best obtainable optical parts mounted on a

[10] Letter of A. A. Michelson to Simon Newcomb, Aug. 15, 1883, printed in *Astr. Papers, U. S. Nautical Almanac Office*, **II**, 248 (1891).

[11] F. Arago, *Compt. Rend.* **7**, 954 (1838); H. Fizeau and L. Breguet, *Compt. Rend.* **30**, 773 (1850); L. Foucault, *Compt. Rend.* **30**, 551 (1850).

[12] A. A. Michelson, *Astr. Papers, U. S. Nautical Almanac Office* **II**, 256–258 (1891).

[13] Lord Rayleigh; *Nature* **24**, 382 (1881); **25**, 52 (1881); J. Willard Gibbs, *Am. J. Sci.* **31**, 62 (1886). Gibbs also showed that the group velocity measured by Michelson in distilled water should lead to a velocity ratio of 1.35 instead of 1.33 found by Michelson. However, the experimental uncertainty in Michelson's determination appears to be ± 0.03 so that his result is in agreement with theory. It was not until 1944 that any notable improvement was made on these early measurements of Michelson's. See R. A. Houstoun, *Proc. Roy. Soc. of Edin.* **62**, 58 (1944). See also a very complete discussion of this subject by Raymond T. Birge, *Lond. Phys. Soc. Reports on Progress in Physics* **8**, 93–94 (1941).

[14] *British Association Report*, 654 (1884).

[15] H. Fizeau, *Annales de Chimie* **57**, 385 (1859).

[16] A. A. Michelson and E. W. Morley, *Am. J. Sci.* **31**, 377 (1886).

[17] H. A. Lorentz, *Arch. Néerl.* **21**, 103 (1886).

rigid stone slab 150 centimeters square and 30 centimeters thick which was floated on mercury. A perspective drawing of the Michelson-Morley interferometer is shown in Fig. 2. Light from an argand burner (not shown) was divided into two pencils which were reflected from four mirrors mounted at each corner of the stone. The reunited pencils were observed in a telescope mounted on the nearby left-hand corner of the stone as shown in the illustration. A wooden cover protected the interferometer from air currents and too-rapid temperature changes. With this apparatus having effective optical paths in each arm of 1100 centimeters, they expected to find an easily measurable shift in the position of the white light interference fringes when the instrument was rotated through an angle of 90° in the horizontal plane. The final observations with this equipment were made in July of 1887 and contrary to all expectations no shift in the interference fringes was observed.[18] At first the significance of this now famous experiment was not appreciated and many early attempts at its explanation are now known to be irrelevant. Even Michelson himself in giving an address on optical research as Vice President of section B of the A.A.A.S. in Cleveland in 1888 failed to mention this result.[19] In fact as late as 1900 Lord Kelvin in his address to the International Congress of Physics in Paris could only refer to the Michelson-Morley experiment as one of the "two small clouds" (the other being the lack of an adequate theoretical explanation for the blackbody spectrum) which dimmed the otherwise brilliant sky of the new scientific century. It was only after the pioneer work of Lorentz and Fitzgerald was followed by Einstein's development of the special theory of relativity in 1905 that the Michelson-Morley experiment received its proper praise and clear explanation.

The following year 1888–1889 was Michelson's last at Case and during this period he began his study of the profiles of spectrum lines with the interferometer, having already made the very important discovery that the red hydrogen line is a doublet.[20] He also laid plans to calibrate the standard meter in terms of light waves. In fact the principal part of his A.A.A.S. address referred to above[19] concerned "A Description of Apparatus for Making the Light Wave the Standard of Length." This work was also carried on in collaboration with Professor Morley and plans were made to measure the standard meter in Paris in terms of light waves by the interferometer technique. The selection of the red cadmium line followed the work in Cleveland, but it was there that the "visibility of the interference fringes" was established as the criterion for the sharpness of spectrum lines.[21] The final work on the standard meter Michelson accomplished at Paris at the International Bureau of Weights and Measures in 1892–1893.

Michelson resigned from the Case Faculty in 1889 and went first to Clark University and then to The University of Chicago. During his long career at the latter institution he added to his fame by extensive researches in spectra and the fine structure of spectrum lines to which work he contributed not only refinements in the interferometer techniques begun in Cleveland, but also the echelon spectrograph (1898), and improved methods for the production of ruled gratings. Also at the University of Chicago in collaboration with Dr. Henry Gordon Gale he performed the earth-tide experiment (1916) on the grounds of the Yerkes observatory at Lake Geneva, and in 1925 measured the rotation of the earth by interferometer observations on light moving in a large rectangular system of evacuated pipes near the present site of the Chicago airport. Finally, in his later years, which he spent largely in California, he improved his velocity of light measurements and at the Mount Wilson Observatory used an interferometer technique first attempted at Yerkes to determine the diameters of red giant stars.

He died on May 9, 1931, the first American to have received the Nobel Prize in physics, the second president of the American Physical Society, and the acknowledged leader in the field of precise optical measurements.

I am greatly indebted to Dr. Karl K. Darrow who suggested that this paper be written and who gave valuable criticisms, and also to Professor Raymond T. Birge for helpful discussions especially relating to the work on the velocity of light.

[18] A. A. Michelson and E. W. Morley, *Am. J. Sci.* **34**, 333 (1887); *Phil. Mag.* **24**, 449 (1887); *J. de Physique* **1**, 444 (1888); *Sidereal Messenger* **6**, 306 (1887).

[19] Article in *The Cleveland Leader*, Aug. 16, 1888.

[20] A. A. Michelson and E. W. Morley, *Am. J. Sci.* **34**, 427 (1887).

[21] A. A. Michelson and E. W. Morley, *Am. J. Sci.* **38**, 181 (1889); *Phil. Mag.* **24**, 463 (1887); *Science* **10**, 86 (1887).

Optical Methods for the Determination of Flame Temperatures*

II. Reversal Methods for Nonisothermal Flames, Two-Path Method, Compensated Hot-Wire Method, Methods Based on Measurements of Line Intensities**

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5. Temperature Measurements on Nonisothermal Combustion Flames

A. Historical Introduction.—All the optical techniques for measuring flame temperatures permit only limited resolution of the flame zone and therefore yield measurements corresponding to effective temperatures over nonisothermal regions of varying extent. In view of this limitation it is apparent that auxiliary considerations are required to permit the correlation of temperature inhomogeneities with experimentally determined results. Preliminary experimental and theoretical work on this problem was reported by Griffiths and Awberry in 1929.[35] However, a thorough study of temperature determinations on complex flames does not appear to have been carried out until very recently.[65] Considerations of temperature measurements on complex flames appear to have been largely restricted to application of the line reversal method.

B. Temperature Measurements through Two Isothermal Flame Regions Which Are at Different Temperatures.—Griffiths and Awberry[35] reported that measured flame temperatures were apparently independent of the thickness of the flame. They attributed this observation, which should be approximately valid for an isothermal flame region, to the simultaneous variation of absorption and emission resulting in an apparent uniform temperature. For nonuniform flame zones the temperature is not independent of thickness as can be demonstrated by an elementary argument.[35, 57, 65]

In order to illustrate the problems which arise when the line reversal method is used for the determination of effective flame temperatures through nonisothermal regions, we may consider the simplest possible case in which the light from a blackbody source at temperature T_S passes first through a flame of thickness x_1, mass absorption coefficient k_1, density ρ_1, emissivity ϵ_1, at the temperature T_1 and then through a flame of thickness x_2, mass absorption coefficient k_2, density ρ_2, emissivity ϵ_2, and temperature T_2. At the point of reversal for this composite flame it is apparent that the following relation must be valid:

$$J(\lambda, T_S) = J(\lambda, T_S)e^{-k_1\rho_1x_1}e^{-k_2\rho_2x_2} + J(\lambda, T_1)(1-e^{-k_1\rho_1x_1})e^{-k_2\rho_2x_2} + J(\lambda, T_2)(1-e^{-k_2\rho_2x_2}), \quad (15)$$

where use has been made of Kirchhoff's law. The effective blackbody temperature $T_{1,2}$ of the composite flame is defined through the relation

$$J(\lambda, T_{1,2}) = J(\lambda, T_1)(1-e^{-k_1\rho_1x_1})e^{-k_2\rho_2x_2} + J(\lambda, T_2)(1-e^{-k_2\rho_2x_2}). \quad (16)$$

From Eqs. (15) and (16) it follows that

$$J(\lambda, T_S)/J(\lambda, T_{1,2}) = [1-\exp(-k_1\rho_1x_1-k_2\rho_2x_2)]^{-1}. \quad (17)$$

Since

$$J(\lambda, T_S)/J(\lambda, T_{1,2}) = \exp\{(c_2/\lambda)[(1/T_{1,2})-(1/T_S)]\},$$

it follows from Eq. (17) that the apparent temperature $T_{1,2}$ of a flame consisting of two isothermal regions depends on the thickness of the isothermal zones and on their optical densities but is apparently independent of the order in which the flame zones are arranged. The interpretation of this conclusion requires some caution since it can be seen from Eq. (16) that the definition of $T_{1,2}$ itself depends on the order in which the flames are arranged and that therefore the effective temperature measured experimentally does depend on the relative order in which the isothermal regions are arranged. Thus, Griffiths

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* Sections, references, and equations are a continuation of Part I which appeared in *Am. J. Physics* **17**, 422 (1949).

** This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under U. S. Army Ordnance Department Contract No. W-04-200-ORD-1482.

[65] F. P. Bundy and H. M. Strong, Project Squid, Technical Memo. No. NYU-2, April, 1948. General Electric Co., Project Hermes, Report No. 55275, Sept., 1948.

and Awberry[35] found that the observed reversal temperature approximated the temperature of the hotter region if the light from the comparison radiator passed first through a cool flame and then through a relatively hot flame. With the reverse arrangement the observed temperature tended to be intermediate between the temperatures of the two flames.

Recently Bundy and Strong[65] have devised a method for the reconstruction of the temperature pattern of a flame composed of two isothermal regions. One of their proposed methods involves application of the reversal equations to one of the component isothermal zones and to the composite flame examined as a whole. Thus the reversal temperature of the blackbody comparison source is T_{S1} at the point of reversal of the isothermal flame region whose temperature is T_1. Hence

$$J(\lambda, T_{S1}) = \alpha_1 J(\lambda, T_1) + J(\lambda, T_{S1})(1-\alpha_1), \quad (18)$$

where $\alpha_1 = \epsilon_1 = 1-\exp(-k_1\rho_1 x_1)$ represents the absorptivity (which is equal to the emissivity by Kirchhoff's law) of the flame whose temperature is T_1. Similarly, at the point of reversal for two isothermal flame regions of temperatures T_1 and T_2, respectively, the comparison source reads a temperature $T_{S1,2}$. Therefore,

$$J(\lambda, T_{S1,2}) = \alpha_2 J(\lambda, T_2) + \alpha_1 J(\lambda, T_1)(1-\alpha_2) + J(\lambda, T_{S1,2})(1-\alpha_1)(1-\alpha_2), \quad (19)$$

where $\alpha_2 = 1-\exp(-k_2\rho_2 x_2)$. Using the relation defining $T_{1,2}$ as given in Eq. (16) it can be seen that Eq. (19) reduces to

$$J(\lambda, T_{S1,2})(\alpha_1+\alpha_2-\alpha_1\alpha_2) = J(\lambda, T_{1,2}). \quad (20)$$

From Eqs. (18) and (20) it follows that

$$J(\lambda, T_{S1})/J(\lambda, T_{S1,2}) = [\alpha_1 J(\lambda, T_1)/J(\lambda, T_{1,2})](1-\alpha_2+\alpha_2/\alpha_1). \quad (21)$$

By the use of Wien's equation, Eq. (21) can be rewritten as

$$\exp\{(c_2/\lambda)[(1/T_{S1,2})-(1/T_{S1})+(1/T_1)-(1/T_{1,2})]\} = \alpha_1(1-\alpha_2)+\alpha_2. \quad (22)$$

The quantity α_2 is given from Eq. (16) as

$$\alpha_2 = \frac{\alpha_1 J(\lambda, T_1) - J(\lambda, T_{1,2})}{\alpha_1 J(\lambda, T_1) - J(\lambda, T_2)}. \quad (23)$$

From the condition of reversal for a single isothermal flame zone it has been shown (see Sec. 3) that

$$T_{S1} = T_1,$$

whence Eq. (22) becomes

$$\exp\{(c_2/\lambda)[(1/T_{S1,2})-(1/T_{1,2})]\} = \alpha_1(1-\alpha_2)+\alpha_2. \quad (24)$$

By using the reversal condition for the flame whose temperature is T_2 it can similarly be shown that

$$\exp\{(c_2/\lambda)[(1/T_{S1,2})-(1/T_{1,2})]\} = \alpha_2(1-\alpha_1)+\alpha_1. \quad (25)$$

It is apparent from Eq. (23) and Eqs. (24) and (25) that the temperature T_2 can be determined from the experimentally determined values of $T_{S1,2}$ provided T_1 and α_1 or α_2 are known. In this manner it is, therefore, possible to ascertain the temperature of one of two isothermal regions from measurements on the other isothermal region and on the composite flame.[66] In practice it is possible that T_1 may be so low that a thermocouple measurement is feasible, thereby supplementing the temperature determination of the composite flame by the reversal method.

Strong and Bundy[65] have made a thorough study of the application of intensity measurements on the sodium D-line contour to a determination of absorptivities and reconstruction of the spectral brightness of two isothermal flame zones. This experimental procedure permits calculation of the unknown temperature T_2 from the simple expression

$$\frac{\alpha_1 J(\lambda, T_1)}{J(\lambda, T_{1,2})}\left(1+\frac{\alpha_2}{\alpha_1}-\alpha_2\right) = \exp\{(c_2/\lambda)[(1/T_{1,2})-(1/T_1)]\}. \quad (26)$$

The procedure of Strong and Bundy requires a rather elaborate experimental arrangement since it involves a quantitative study of line intensity measurements on the contours produced by self-absorption of the sodium D-line during passage through the cooler flame region.

[66] In principle it is, of course, possible to extend this technique of the study of complex flames with the result that the temperature of the nth zone can be inferred from measurements on the composite flame if the temperatures of $n-1$ zones have been determined independently.

It is apparent from the foregoing discussion that the difficulties of determining even the temperature of one of two isothermal regions without making a direct determination on this region are such as to limit seriously the practical value of measurements of this type since flames in which two well-defined isothermal regions exist may be difficult to approximate.

For a description of the experimental arrangement used for the determination of temperatures on composite flames and for a more complete discussion of the theory the reader is referred to the published literature. Bundy[65] has made a theoretical study of line widths of the sodium line in order to facilitate interpretation of experimental results. The interpretation of temperature measurements made by use of the two-path method (see Sec. 6) if the flame consists of two concentric spherical isothermal regions has also been considered.

It may be of interest to note in this connection that detailed studies of the effect of line width on experimental results were made some 40 years ago.[67] A recent and thorough treatment of the self-absorption of spectral lines may be found in an article by Cowan and Dieke.[68]

6. The Two-Path Method

The two-path method for the measurement of flame temperatures is an optical technique which permits temperature measurements on systems in which the intensity of radiation emitted varies rapidly (and aperiodically) with time. Flame temperature measurements by use of the techniques described previously are generally unsuitable for applications of this type. For example, the reversal methods could be employed only if the comparison radiator also emits radiation the intensity of which varies with time in the same proportion as in the flame under study. The two-path method has been described by Hottel and Broughton[27] and is finding application at the present time to temperature determinations on pulse-jet flames and other systems in which rapid temperature fluctuations occur.[69, 70]

The two-path method depends upon a comparison of (spectral) brightness temperatures when two different path lengths through a flame are viewed separately. In practice it may prove to be desirable to make measurements at wavelengths corresponding to emission from an added substance which is not normally present in the flame. The sodium D-line has been employed for this purpose.[69] Other investigators recommend use of the 4.4μ-emission band of carbon dioxide[70] or of a narrow wavelength band in the visible region[70] which can be isolated through the use of interference filters[71, 72] and detected by means of photocells. It is evident that for the two-path method, as for other optical methods for measuring flame temperature, it is essential that the substance whose radiant emission is studied is in statistical equilibrium with the random translational velocity distribution of the gas molecules.

In the two-path method the path length can be conveniently doubled by placing a mirror at one end of the field of view and making appropriate corrections for the reflectivity of the mirror. It is apparent that a rather uncertain correction for (multiple) reflection from the chamber walls should also be introduced. If the motor wall acts as a cool blackbody, then the radiant energy emitted by a path length L of the combustion region, neglecting transmission losses and the like through windows, in the wavelength interval between λ and $\lambda+d\lambda$, is

$$J(\lambda, T_{br1})d\lambda = J(\lambda, T_F)[1-\exp(-k_\lambda \rho L)]d\lambda, \quad (27)$$

where $J(\lambda, T_{br1})$ is the observed radiant energy, T_{br1} the apparent brightness temperature corresponding to a path length L, $J(\lambda, T_F)$ the radiant energy emitted by a blackbody at the true flame temperature T_F, k_λ the spectral mass emissivity, and ρ the density of the gaseous emitters. When the path length is doubled by placing a mirror at one end of the field of view, the observed radiant intensity corresponding to the apparent brightness temperature T_{br2} is

$$J(\lambda, T_{br2})d\lambda = J(\lambda, T_F)[1-\exp(-k_\lambda \rho L)] \times [1+R_\lambda \exp(-k_\lambda \rho L)]d\lambda, \quad (28)$$

where R_λ is the spectral reflection coefficient of

[67] R. Ladenburg and F. Reiche, *Ann. der Physik* **42**, 181 (1913).
[68] R. D. Cowan and G. H. Dieke, *Rev. Mod. Physics* **20**, 418 (1948).
[69] J. H. Hett, Project Squid, Technical Memo. NYU-2, April, 1948.
[70] J. A. Sanderson, J. A. Curcio, and D. V. Estes, Project Squid, Technical Memo. NYU-2, April, 1948.
[71] J. A. Anderson and R. L. Ueberall, NRL Report H-2731, January, 1946.
[72] L. N. Hadley and D. M. Dennison, *J. Opt. Soc. Am.* **37**, 451 (1947).

the front-surface reflecting mirror. From Eqs. (27) and (28) it follows that

$$J(\lambda, T_{br2})/J(\lambda, T_{br1}) = 1 + R_\lambda \exp(-k_\lambda \rho L). \quad (29)$$

From Eq. (29) it can be seen that the intensity ratio $J(\lambda, T_{br2})/J(\lambda, T_{br1})$ approaches $1+R_\lambda$ as $k_\lambda \rho L$ approaches zero whereas the intensity ratio tends toward unity as $k_\lambda \rho L$ becomes large. Reference to Eq. (27) shows that

$$\exp(-k_\lambda \rho L) = 1 - J(\lambda, T_{br1})/J(\lambda, T_F),$$

whence

$$1 - J(\lambda, T_{br2})/J(\lambda, T_{br1}) = -R_\lambda [1 - J(\lambda, T_{br1})/J(\lambda, T_F)], \quad (30)$$

or

$$\{1 - \exp[(-c_2/\lambda)(1/T_{br2} - 1/T_{br1})]\} = -R_\lambda \{1 - \exp[(-c_2/\lambda)(1/T_{br1} - 1/T_F)]\}, \quad (30a)$$

where c_2 is the second radiation constant and the radiant intensities $J(\lambda, T)$ have been replaced by the use of Wien's equation. It is apparent from Eqs. (30) and (30a) that the experimental determination of $J(\lambda, T_{br1})$ and $J(\lambda, T_{br2})$ permits calculation of $J(\lambda, T_F)$ and, therefore, determines the gas temperature T_F uniquely according to Planck's (or, to a close approximation, Wien's) equation.

The preceding summary of the principles involved in flame temperature measurements by use of the two-path method has been restricted to an isothermal region as source of radiation. As is also the case with other optical methods for flame temperature measurements, considerable complication is introduced whenever measurements are made on nonisothermal regions (see Sec. 5). Bundy[65] has considered the utility of the two-path method if the flame on which measurements are made consists of a hot flame zone surrounded by a cooler region. His idealized calculations led him to conclude that the two-path method would yield reasonable results (corresponding closely to the actual temperature of the hot central core of the flame) if the product $k_\lambda \rho L$ was small enough so that the intensity ratio $J(\lambda, T_{br2})/J(\lambda, T_{br1})$ as given by Eq. (29) would be of the order of $\sqrt{2}$ if R_λ is equal to unity.

7. The Compensated Hot-Wire Method

A. Historical Introduction.—A compensated thermocouple technique for flame temperature measurements was employed by Berkenbusch in 1899 at the suggestion of W. Nernst.[72] Berkenbusch's method effectively eliminates the troublesome radiation correction required for flame temperature measurements made with thermocouples. Schmidt[36] replaced the thermocouple used by Berkenbusch by a hot-wire device. A method practically identical with that used by Schmidt was employed some years later at the National Physical Laboratory[74] and also by Griffiths and Awberry,[35] who do not appear to have been aware of the earlier work of Schmidt. Schmidt,[36] Griffiths and Awberry,[35] and Kohn[37] found excellent agreement between results obtained by the compensated hot-wire technique and by the line-reversal method. More recent discussions of the compensated hot-wire method have been given, for example, by Lewis and von Elbe[43] in a review article on the measurement of flame temperature.

B. The Hot-Wire Method.—The hot-wire method for the measurement of flame temperatures is included in the present survey because it is probably the simplest and most useful non-optical technique for checking or supplementing results obtained through application of one of the optical methods. The hot-wire method is based on an energy balance for a radiating wire placed in a vacuum or exposed to a flame. Schmidt[36] heated an extended wire electrically in a vacuum, and determined its temperature with an optical pyrometer. The rate of loss of energy by radiation per unit length of wire E was measured simultaneously in order to determine the functional relation between E and the wire temperature T. Next the rate of energy input E' into the wire required to raise it to the temperature T' while it was exposed to the flame zone was determined. At the point of intersection of a plot of E *vs.* T with a plot of E' *vs.* T', it is evident that $T=T'$ and $E=E'$. But the condition $T=T'$ and $E=E'$ assured the existence of thermal equilibrium[75] between the wire and the flame since the wire was losing energy by radiation at the same rate in the flame as in a vacuum while it was at the same temperature in both.

[72] F. Berkenbusch, *Ann. der Physik u. Chem.* **67**, 649 (1899).
[74] Report for 1926, p. 63.
[75] Equilibrium exists only if the opacity of the flame for the energy radiated by the wire is negligibly small. This condition seems to be fulfilled in practice in many cases.

The method of Griffiths and Awberry[35] is identical with that used by Schmidt except that the current input into the wire is measured instead of the total energy lost by radiation. If I represents the current required to heat the wire to the temperature T in a vacuum, and I' is the current required to heat the wire to the temperature T' in the combustion zone, then the condition $I=I'$ at $T=T'$ again assures the existence of thermal equilibrium between the wire and the flame. Since it is easier to measure the current input into a wire than to determine the energy lost by radiation from a wire, it seems preferable to employ the experimental arrangement of Griffiths and Awberry rather than that of Schmidt.

C. Limitations of the Compensated Hot-Wire Method.—It is apparent that the compensated hot-wire method will provide a measure of the true gas temperature only if the wire does not by its presence disturb the combustion processes. Furthermore, the wire must retain the same emissivity per unit length in the flame as it possessed in a vacuum. That the wire does, in general, cause some disturbance of the combustion process has been recognized for some time.[37] Pitting of the wire surface also appears to be a source of error in some cases.[36] The hot-wire technique is thus seen to involve some of the limitations of thermocouple measurements and probe techniques. As is the case with thermocouple measurements, a point-by-point exploration of the flame region can be made. Compared with thermocouple measurements, the compensated hot-wire technique has the advantage of eliminating the radiation corrections which must be made whenever thermocouples are used at elevated temperatures. Furthermore, since only a single wire is used, no error can be introduced by preferential catalysis of the combustion process by one or the other of the component wires of a thermocouple as used ordinarily or of the compensated thermocouple introduced by Berkenbusch.[73]

8. Estimation of Flame Temperature from the Change in Spectral Line Intensity with Quantum Number

A. Historical Introduction.—Flame temperature measurements utilizing the change in intensity of spectral lines with quantum number were introduced by Ornstein and his collaborators in 1928. Ornstein and van Wyk[76–78] used the O−O band of the negative nitrogen bands observed during electric discharges in order to determine the rotational temperature. The determination of the rotational temperature yields, of course, a significant result only if the rotational energy levels are in statistical equilibrium with the random translational molecular energies or at least behave as though this statistical equilibrium were maintained. Lindh[79] showed that the distribution of rotational levels in nitrogen corresponded to statistical equilibrium during electronic collision, in agreement with earlier findings by Harries[80] who noted that electron impact does not appreciably alter the rotational distribution. Elliott[81] found a Boltzmann distribution of rotational levels corresponding to the temperature of active nitrogen during activation with active nitrogen. Temperature determinations based on the change in observed intensity for the vibrational levels of the $^2\Sigma$——$^2\Sigma$-bands of cyanogen during an electric discharge were found by Ornstein and Brinkman[82] to be in agreement with rotational temperatures. Temperature determinations on electric arcs using the rotational intensity variation have also been made on the spectra of gold[83] and hydrogen[84] with good results. The early work on temperature measurements in arcs has been reviewed by Ornstein[85] who, incidentally, calls attention to the fact that different rotational temperatures were obtained for the bands of cyanogen and aluminum oxide, a fact which he attributes to emission from different positions in the arc (which are at different temperatures) for the two emitting molecular species.

[76] L. S. Ornstein and W. R. van Wyk, *Zeits. f. Physik* **49**, 315 (1928).
[77] W. R. van Wyk, *Proc. Amsterdam Acad. Sci.* **32**, 1243 (1929).
[78] G. Herzberg, *Molecular spectra and molecular structure I, diatomic molecules* (Prentice-Hall, Inc., New York, 1939).
[79] A. E. Lindh, *Zeits. f. Physik* **67**, 67 (1931).
[80] W. Harries, *Zeits. f. Physik* **42**, 26 (1927).
[81] A. Elliott, *Zeits. f. Physik* **67**, 75 (1931).
[82] L. S. Ornstein and H. Brinkman, *Proc. Amsterdam Acad. Sci.* **34**, 33 (1931).
[83] L. S. Ornstein and S. Sambursky, *Proc. Amsterdam Acad. Sci.* **34**, 339 (1931).
[84] L. S. Ornstein, J. G. Eymers, and J. Wouda, *Proc. Amsterdam Acad. Sci.* **34**, 505 (1931).
[85] L. S. Ornstein, *Physikalische Zeits.* **32**, 517 (1931).

Shortly after the pioneering work of Ornstein and his collaborators had been reported, it became apparent that the rotational line intensity method may lead to erroneous values for the temperature because of abnormal rotational energy distributions. Gaviola and Wood[86] found an abnormal population of the higher rotational energy levels of HgH as the result of excitation by Hg, a phenomenon which has since been studied extensively by Rieke[87] who observed a more nearly normal rotational distribution of HgH in the presence of OH than in the presence of N_2. Abnormal rotation appears in the negative bands of CO and the negative bands of N_2^+ excited in a low voltage arc,[88] whereas a more nearly normal distribution of energy is observed for the same bands in the electrodeless discharge and in a neon transformer glow discharge. Abnormal excitation of rotational levels has been observed for C_2 and CH during electric discharges through acetylene,[89] for OH during an electric discharge through water,[90] and for CuH in an electric arc.[91] The abnormal rotation of OH has been the subject of extensive study by Oldenberg[92] who found normal rotation of OH formed in an electric discharge through H_2O during absorption and noted that the abnormal rotation observed during emission could be greatly reduced by the addition of large amounts of helium. More recently Lyman[93] has made temperature determinations using the $^2\Sigma$——$^2\Pi$-transition of OH in oxygen-hydrogen flames and in copper arcs. She found a thermal distribution of intensities from which temperatures of 3275°K and 6350°K, respectively, were calculated. The OH spectrum in a vacuum tube discharge showed abnormal rotation which was found to be dependent on the pressure of water vapor in the tube. A comparison of spectroscopically determined rotational temperatures in arcs and of thermometric temperatures has been given by Horie.[94]

As is evident from the preceding literature survey of some of the work on temperature determinations by means of the rotational or vibrational line intensity variation, most of the applications of this technique have been concerned with studies on arcs. Oldenberg[92] has given a discussion of some of the factors responsible for the production of abnormal rotation in molecules which will be considered later on with reference to their applicability to combustion flames. The occurrence of abnormal vibrational or rotational excitation in combustion flames cannot be ruled out *a priori* in view of some direct experimental findings of vibrational (and possibly of rotational) time lag of O_2 and N_2 during explosions in a spherical vessel.[95] More recently Gaydon noted a significant change in the rotational temperature of OH produced as the result of the addition of acetylene.[96]

B. Principles Involved in Flame Temperature Measurements Based on the Variation in the Intensity of Spectral Lines with Quantum Number.— The principles involved in flame temperature determinations by use of the variation in the intensity of spectral lines are given, for example, in one of the well-known textbooks on molecular spectra by Herzberg.[78] For the present purposes a summary of the relevant formulas for determining the intensity of spectral lines, similar to that given recently by Chwalow,[97] seems in order.

Since quadrupole transition probabilities for radiation of visible light are only about 10^{-8} times the dipole transition probabilities, the following discussion will be restricted to dipole radiation because of the obvious difficulty of observing the weak emission or absorption lines corresponding to quadrupole radiation. For reasons which have been discussed by Oldenberg,[92] reliable temperature measurements are more

[86] E. Gaviola and R. W. Wood, *Phil. Mag.* **6**, 1191 (1928); E. Gaviola, *Phil. Mag.* **6**, 1167 (1928); *ibid.* **6**, 1154 (1928).

[87] F. F. Rieke, *Physical Rev.* **42**, 587 (1932); *J. Chem. Physics* **4**, 513 (1936) and **5**, 831 (1937).

[88] O. S. Duffendack, R. W. Revans, and A. S. Roy, *Physical Rev.* **45**, 807 (1934).

[89] W. Lochte-Holtgreven, *Zeits. f. Physik* **64**, 443 (1930); *ibid.* **67**, 590 (1931).

[90] K. F. Bonhoefer and T. G. Pearson, *Zeits. f. Physikalische Chem.* (B) **14**, 1 (1931).

[91] H. Schüler and H. Gollnow, *Zeits. f. Physik* **108**, 714 (1938); *ibid.* **109**, 432 (1938). The anomalous excitation of CuH has recently been ascribed to a catalytic effect produced by the surface of the copper cathode: L. Reinebeck, *Zeits. f. Naturforschung* **2a**, 251 (1947).

[92] O. Oldenberg, *Physical Rev.* **46**, 210 (1934); *J. Chem. Physics* **3**, 272 (1935).

[93] E. R. Lyman, *Physical Rev.* **53**, 379 (1938).

[94] T. Horie, *Proc. Physico-Math. Soc. of Japan* (3), **22**, 787 (1940).

[95] B. Lewis and G. von Elbe, *Combustion flames and explosions of gases* (Cambridge University Press, 1938).

[96] A. G. Gaydon and H. G. Wolfhard, *Proc. Roy. Soc.* **A194**, 169 (1948).

[97] M. L. E. Chwalow, Picatinny Arsenal Report No. 1666, October, 1947.

difficult to make by the use of intensity variations of vibrational lines than of rotational lines (see, however, Ref. 82 for application of the vibration spectrum of cyanogen). Furthermore, since the theoretical considerations are similar, the present discussion will be restricted to the change in intensity of rotational lines.

The intensities of rotational lines during absorption I_{abs} and emission I_{em} are given, to a good approximation, by the following two relations, respectively:[78]

$$I_{abs}=\text{const.}\times(J'+J''+1)\times e^{-B''J''(J''+1)hc/kT}\nu R^2, \quad (31)$$

$$I_{em}=\text{const.}\times(J'+J''+1)\times e^{-B'J'(J'+1)hc/kT}\nu^4 R^2. \quad (32)$$

Here J' is the rotational quantum number of the upper state, J'' the rotational quantum number of the lower state, h is Planck's constant, c is the velocity of light, k represents the Boltzmann constant, T is the absolute temperature, ν is the observed frequency of the spectral lines (which is very nearly constant for a given vibration-rotation band), R is the matrix element corresponding to the given dipole transition, and B' and B'' are defined through the relations $B'=h/(8\pi^2cI')$ and $B''=h/(8\pi^2cI'')$ where I' and I'' are the moments of inertia corresponding to the upper and lower states, respectively. The selection rules for various spectral transitions are dependent upon the vanishing or nonvanishing of the matrix element $\mathbf{R}^{nm}$ between two states n and m. The vector $\mathbf{R}^{nm}$ is defined through the relation

$$\mathbf{R}^{nm}=\int\psi_n\psi_m^*(\sum_i e_i\mathbf{r}_i)d\gamma,$$

where ψ_n and ψ_m are the respective eigenfunctions of the two combining states. The star denotes the complex conjugate, $\sum_i e_i\mathbf{r}_i$ is the total dipole moment (i.e., the dipole moment summed over all of the charged particles) with components $p_x=\sum_i e_ix_i$, $p_y=\sum_i e_iy_i$, $p_z=\sum_i e_iz_i$, and the integral is extended over the $3N$-dimensional configuration space of the N particles composing the molecule whose rotational intensities are being determined.[78] The squares of the matrix elements correspond to the classical transition probabilities which can be evaluated either by the use of Einstein transition probabilities[98] or from Van Vleck's correspondence principle for absorption.[99] For a more satisfactory treatment of absorption and emission of radiation, reference should be made to the work of Dirac.[100]

For a given vibration-rotation band the factors νR^2 and ν^4R^2 are very nearly independent of the rotational energy levels whence Eqs. (31) and (32) reduce to Eqs. (33) and (34), respectively:

$$\log\frac{I_{abs}}{J'+J''+1}=\log C-\frac{B''J''(J''+1)hc}{kT}, \quad (33)$$

$$\log\frac{I_{em}}{J'+J''+1}=\log C'-\frac{B'J'(J'+1)hc}{kT}, \quad (34)$$

where $C=\text{const.}\times\nu R^2$ and $C'=\nu^4C$ are, approximately, independent of the factor $J(J+1)$. Therefore, a plot of $\log I_{abs}/(J'+J''+1)$ *vs.* $J''(J''+1)$ or of $\log I_{em}/(J'+J''+1)$ *vs.* $J'(J'+1)$ should be linear with the slope yielding the value of the rotational temperature.[78]

Equations (33) and (34) refer to $^1\Sigma$——$^1\Sigma$-transitions involving states with zero electronic angular momentum. For a discussion of other transitions reference should be made to Herzberg[78] or to the quantitative treatment of Hönl and London.[101] A discussion of refinements of Eqs. (33) and (34) taking into account such factors as vibration-rotation interactions can be found in the literature.[78]

C. Application of the Variation of Rotational Line Intensity with Quantum Number to Flame Temperature Measurements.—For temperature determinations in combustion gases the investigator has usually a choice of molecular species which can be used for temperature measurements. For example, in a gasoline-oxygen flame, temperature determinations can be made, in principle, from the spectra of OH, C_2, CH, CO, O_2, and N_2. The choice of molecular species becomes thus a matter of convenience provided the rotational energy distribution of the selected molecular species fulfills the requirement of being in statistical equilibrium with the random translational energy of the gas molecules.

[98] A. Einstein, *Physikalische Zeits.* **18**, 121 (1917); *Verh. der Deuts. Physikalische Ges.* **18**, 318 (1916).
[99] J. H. Van Vleck, *Physical Rev.* **24**, 330, 347 (1924).
[100] P. A. M. Dirac, *Proc. Roy. Soc.* **A112**, 661 (1926); *ibid.* **A114**, 243 (1927).
[101] H. Hönl and F. London, *Zeits. f. Physik* **33**, 803 (1925).

Since the theoretical treatments of temperature measurements from absorption and from emission spectra are similar, there seems to be little preference for one or the other method from this point of view. However, attention should be called to the fact that emission spectra are much more intense than absorption spectra and are therefore more easily observed. The reason for this is the proportionality of I_{em} to ν^4 rather than to ν as is the case for I_{abs} (see Eqs. (31) and (32)).

Oldenberg[92] has discussed abnormal rotation in molecules and has given a review of some of the processes which may be responsible for abnormal rotational excitation. The processes recognized by Oldenberg are the following:

(1) Simultaneous dissociation and excitation of polyatomic molecules. The production of abnormally excited OH (designated as OH*) during electric discharges through H_2O is a well known example of this type and may be described schematically as

$$\text{energy}+H_2O\rightarrow H+OH^*.$$

Oldenberg's work, proving that the addition of helium, which is known not to alter electronic excitation,[102] produced a more nearly normal emission spectrum of OH, would appear to indicate no persistence of abnormal rotation during collision.[103]

(2) Excitation of diatomic molecules may result in abnormal vibration in accord with the Franck-Condon principle. Some of the excess energy may then be transferred to rotational energy levels as the result of impacts during the life of the excited vibrational state. Rieke[87] has shown that this process is responsible for abnormal excitation of HgH. Oldenberg suggests that the intensity change in certain H_2 bands produced by the addition of He may have a similar origin.[92]

(3) Excitation by impact. Oldenberg cites two examples, one of which involves preferential excitation of N_2 by a canal ray.[104]

(4) Abnormal excitation as the result of perturbations. Oldenberg indicates that the fact that different rotational temperatures were found[105] for the strong and weak lines of F_2 may be attributed to perturbations as may also the similar effect[106] observed for CN.

Of the four processes listed by Oldenberg as possible sources of abnormal energy distribution, only the first two would be expected to be of importance for studies on combustion flames. Perhaps the terminology should be changed by replacing Oldenberg's excitation as the result of dissociation by the more inclusive designation of excitation associated with chemical reaction.

The vibrational energy distribution is easily changed as the result of excitation by collision because of the operation of the Franck-Condon principle.[92] This fact forms the basis for the statement made by Pearse and Gaydon[107] that band systems arising from states in which the configuration of the nuclei differs markedly from that of the normal state may be relatively strong in flames but not in arcs since excitation in flames occurs mainly as the result of collisions. Abnormal excitation of vibrational levels may then lead to abnormal rotational energy distribution in accord with Oldenberg's mechanism (2) listed above. However, it should be noted that energy exchange by collision will ultimately be responsible for a thermal distribution of energy. It follows, therefore, that at elevated pressures the rate of exchange of energy by collision may be so rapid that a thermal energy distribution is maintained at all times during combustion, for all practical purposes.

The rate of attainment of thermodynamic equilibrium after the occurrence of abnormal excitation can be analyzed, approximately, in terms of classical physical considerations. The exchange of energy by collision will evidently be limited because of the requirements of the laws of conservation of energy and momentum.[108] Therefore, progress toward equilibrium will be slow for col-

[102] L. von Hamos, *Zeits. f. Physik* **74**, 379 (1932).
[103] H. Beutler and E. Rabinowitsch, *Zeits. f. Physikalische Chem.* (B) **8**, 403 (1930).
[104] H. D. Smyth and E. G. F. Arnott, *Physical Rev.* **36**, 1023 (1930).
[105] H. G. Gale and G. S. Monk, *Astrophysical J.* **69**, 77 (1929).
[106] M. Fassbender, *Zeits. f. Physik* **30**, 90 (1924).
[107] R. W. B. Pearse and A. G. Gaydon, *The identification of molecular spectra* (John Wiley and Sons, New York, 1941).
[108] O. Oldenberg and A. A. Frost, *Chem. Rev.* **20** 99 (1937); *Physical Rev.* **37**, 194 (1931).

lisions between particles of greatly different masses. In a combustion chamber operating at elevated pressures it is not to be expected that appreciable excitation anomalies will persist for any length of time because of the frequent occurrence of collisions between many different molecular species, free radicals, and ions. However, the possibility of abnormal excitation in regions of active combustion cannot be ruled out *a priori* because the balance between rates of abnormal excitation by reaction and collision and rates of equalization of energy by collision is not known in sufficient detail to permit useful predictions.

At elevated pressures even the vibrational energy levels of some of the molecules may have population densities corresponding to statistical equilibrium and, therefore, the intensity change of vibrational lines may be suitable for temperature determinations.[82, 109, 110] Nevertheless, caution seems to be in order regarding the application of the line intensity technique for temperature measurements, particularly in view of some experimental findings which indicate abnormal distribution of energy.[95, 96] In addition to the methods described in other sections of this report, valid temperature measurements can frequently be obtained by using line intensity variations in absorption spectra,[92, 111–113] emission spectra excited by heat,[92, 114, 115] or emission spectra of molecules such as N_2, which are not readily excited abnormally.

Since the rate of establishment of thermal equilibrium at low pressures is relatively slow, it is not surprising that examples of abnormal rotational energy distribution in low pressure combustion flames are encountered. A very striking example of processes of this type has been reported by Gaydon[96] who noted a change in the rotational temperature of OH of several thousand degrees when some acetylene was introduced into the flame. Discussions of excitation lags in gases under various other conditions have been reported in the literature. Kneser and Knudsen found excitation lags in experiments with sound waves and have given numerous references to earlier work on the same subject.[116] Large rotational time lags have been reported during rapid compressions of diatomic gases[117] and for the discharge of steam through an orifice.[118] Considerable work on the general subject of excitation lags in gas dynamics has been reported[119–122] and may ultimately prove to be of use for predicting the molecular species which can be used for reliable temperature determinations.

9. Conclusions

The survey of optical methods for the measurement of flame temperature presented in Secs. 1 to 8 has been concerned primarily with a review of important principles involved in temperature determinations. The discussion was based on the assumption that a temperature measurement will be useful, from the operational point of view, only if it is a direct measure of the random translational energy of the molecules. As explained in the Introduction, this point of view was adopted in order to make the temperature a measure of combustion efficiency as defined ordinarily in practical applications. It should perhaps be emphasized that for some purposes, such as the investigation of rotational excitation or the formation of molecules of abnormal energy content, a useful "temperature measurement" would be a temperature measurement under nonequilibrium conditions. Thus if we were interested in determining the distribution of hydroxyl radicals in the various rotational energy levels, then we would measure the intensity of the rotational emission lines and would not be particularly concerned with the equality of the rotational temperature of OH and the mean translational temperature of all of the gas molecules. This problem arises only if our objective is the determination of the mean translational tempera-

[109] G. Herzberg, *Zeits. f. Physik* **49**, 761 (1928).
[110] N. R. Tawde, *Proc. Physical Soc.* **46**, 324 (1934).
[111] H. Kuhn, *Zeits. f. Physik* **39**, 77 (1926).
[112] E. Hutchisson, *Physical Rev.* **36**, 416 (1930).
[113] K. Wurm, *Zeits. f. Astrophysik* **5**, 260 (1932).
[114] L. S. Ornstein, H. Brinkman, and A. Beunes, *Zeits. f. Physik* **77**, 72 (1932).
[115] D. Th. J. Ter Horst and G. Krygsman, *Physica* **1**, 114 (1933).
[116] H. O. Kneser and V. O. Knudsen, *Ann. der Physik* **21**, 682 (1935).
[117] B. Lewis and G. von Elbe, *J. Chem. Physics* **7**, 197 (1939).
[118] B. V. Korvin-Kroukovsky, *J. Franklin Inst.* **227**, 99 (1939).
[119] L. Landau and E. Teller, *Physikalische Zeits. der Sowjetunion* **1**, 34 (1936).
[120] A. Kantrowitz, *J. Chem. Physics* **14**, 150 (1946).
[121] P. W. Huber and A. Kantrowitz, *J. Chem. Physics* **15**, 275 (1947).
[122] C. Zener, Eleventh Report of the Committee on Contact Catalysis, National Research Council, p. 103, 1935.

ture. It is evident that any optical measurement of flame temperature will yield definite and useful information concerning some aspects of the flame and the gases.

A middle-of-the-road course has been followed throughout the present discussion as regards the experimental measurement of the translational temperature. Thus attention has been called to the fact that erroneous temperature readings are obtained in some cases. At the same time an effort has been made to emphasize that the definition of the translational temperature retains its usefulness under all conditions and that this temperature can be measured if not with one method then with another. Unfortunately, it is not possible to give a concise prescription for evaluating the translational temperature under all conditions. An attempt to present a conclusion of this type would be inconsistent not only with the thesis of the present survey but would also conflict with some experimentally observed results.

We do not have *a priori* information concerning the approach to statistical equilibrium in a given combustion flame. In the absence of evidence to the contrary, we are therefore justified in choosing any optical technique for which the equipment is available or which appears particularly interesting for other reasons. In many cases (e.g., the flame of a Meker burner) our first temperature measurement will probably determine the translational temperature. In general, however, it is necessary to verify the equilibrium assumption by demonstrating that concordant results can be obtained by the use of two or more independent techniques. Detailed knowledge of the experimental findings of other investigators will permit the intelligent exclusion of some methods under certain conditions. For example, for a temperature determination in a region of active combustion we might prefer one of the reversal methods to a measurement based on the change of rotational line intensity with quantum number while, at the same time, we would expect absorption-emission pyrometry to be less likely to give large errors than the line reversal method. Nevertheless, it is entirely possible that the translational temperature of the flame under study can be determined equally well by all three methods.

Acoustical Impedance and Absorption Coefficients

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THE determination of the normal coefficient of absorption and acoustical impedance of materials is an important and interesting type of experiment which may be performed in the usual acoustical laboratory by the method of standing waves in a tube.

A loudspeaker may be used to set up standing waves at one end of a tight, heavy tube with the acoustical material at the opposite end as shown in Fig. 1. The normal absorption coefficient is found by measurement of the difference between the maximum H_2 and minimum L_1 intensity in decibels (i.e., H_2-L_1) using a traveling pressure microphone. The resistive and reactive components of impedance are found by measurement of the ratio D_1/D_2 in addition to the above two measurements for the normal absorption coefficient. The distance from the acoustical material to the first excess pressure minimum is D_1, while the distance from the first excess pressure minimum to the second excess pressure minimum is D_2.

Theory

The following formula has been developed[1,2] for the normal coefficient of absorption a_n when

[1] S. L. Brown and M. Y. Colby, *Laboratory manual, electricity and magnetism, light, and sound* (University Cooperative Society, Austin, Texas, 1944), p. 45.
[2] H. J. Sabine, *J. Acous. Soc. Am.* **14**, 143 (1942).

the tube method is used

$$a_n = 1 - \left(\frac{\log_{10}^{-1} L/20 - 1}{\log_{10}^{-1} L/20 + 1} \right)^2 \qquad (1)$$

where $L(=H_2-L_1)$ is the difference between the maximum and minimum intensities in decibels. This equation may also be written as

$$a_n = 1 - k^2 \qquad (2)$$

where k is the pressure reflection coefficient.

Expressions for the resistive R and reactive X components of the acoustical impedance have been derived by Davis,[3] Olsen,[4] and Morse.[5] They are as follows

$$\frac{R}{\rho c} = \frac{1-k^2}{1+k^2+2k\cos(2\pi D_1/D_2)} \qquad (3)$$

and

$$\frac{X}{\rho c} = \frac{2k\sin(2\pi D_1/D_2)}{1+k^2+2k\cos(2\pi D_1/D_2)} \qquad (4)$$

where ρ is the density of the air, c is the velocity of sound in air, k is the pressure reflection coefficient as defined in Eq. (2) and D_1/D_2 is the ratio previously defined and shown in Fig. 1.

For air at sea level the specific acoustic resistance ρc is approximately 42.7 g/(sec cm^2). Thus R and X are found readily from Eqs. (3) and (4), and the acoustic impedance is given by

$$Z_a = R_a + jX_a \qquad (5)$$

with magnitude

$$|Z_a| = (R^2+X^2)^{\frac{1}{2}}. \qquad (6)$$

Procedure

There are two impedance tubes in the acoustic laboratory at William Jewell College. Both are nine feet long with square cross sections and inside areas 1.0 and 0.25 ft^2 respectively. The former is used for absorbing materials such as diaphragmatic tiles that may not be examined in smaller surface units. The latter may be used at much higher frequencies and is, therefore, more useful when materials of area 0.25 ft^2 may be tested. Both tubes are constructed of well-seasoned two-inch lumber which was sanded, covered with wood filler and then given two coats of shellac.

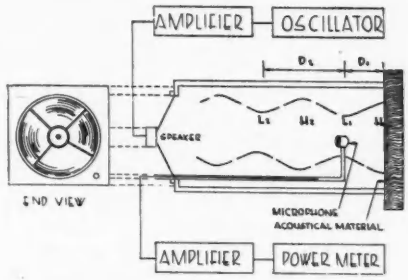

FIG. 1. Arrangement of apparatus used to determine acoustical impedance and absorption coefficients.

Other items of equipment include a variable oscillator, a high quality amplifier, speaker, small traveling microphone of good quality, and some type of output meter such as a power meter, voltmeter, or sound level meter.

The apparatus is arranged as shown in Fig. 1. The sound source consists of a variable oscillator connected to the speaker through an amplifier. The acoustical material is attached to a backing material in the same way as used on the wall and the whole is secured tightly to the tube by the end plate which is bolted to the tube. Vacuum wax or the equivalent may be used to secure tightness if in doubt. The small traveling pressure microphone serves as a detector of the nodes and loops of pressure as it moves up and down the tube on two tiny rollers. The cross-sectional area of about 16.2 percent intercepted by the microphone and roller mechanism does not change the results appreciably. The maximum and minimum intensity of the standing wave pattern as measured by the excess pressure amplitude is read on the output meter and their difference is given by

$$L = H_2 - L_1.$$

The harmonic components of the tube may be computed provisionally by use of the formula,

$$f_m = mc/2l,$$

where c and l represent the velocity of sound and the length of the tube respectively, and m is the order of the harmonic. The second harmonic where $m=2$ represents the lowest tone for which

[3] A. H. Davis, *Modern acoustics* (Macmillan, New York, 1934), p. 262.
[4] H. F. Olsen, *Elements of acoustical engineering* (D. van Nostrand, New York, 1940), p. 384.
[5] P. M. Morse, *Vibration and sound* (McGraw-Hill, New York, 1936), p. 192.

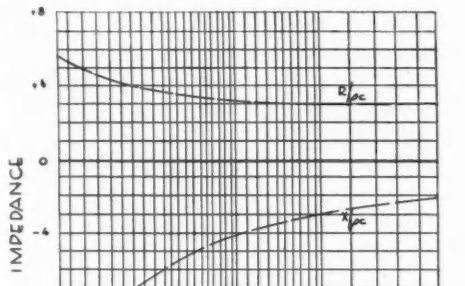

FIG. 2. Resistive and reactive components of acoustical impedance of a single diaphragmatic acoustical tile with cross section of 1.0 ft².

resonant measurements are practical. In our tube the second harmonic has a frequency about 130 cycles per second. Overtones corresponding to frequencies of about 260, 390 and 520 have also been used.

Since there is attenuation within the tube, there is some increase in the intensity at successive minima as one measures these minima away from the sample. In general, L_2 will be greater than L_1, but if the tube is well constructed, the error will not be serious unless high accuracy is desired. In this case, a method of correction is given in Sabine's[2] article.

The real and reactive components of the impedance may be plotted as shown in Fig. 2. The reactive component may decrease in magnitude and pass through the zero value. The real part may be expected to be more nearly straight except for factors not accounted for in the above theory. The absolute value of the impedance may be drawn by some distinguishing curve on the same graph. A graph of the companion normal absorption coefficient coupled with the absolute value of the impedance gives the most useful type of information on acoustical absorption material.

The experiment is important in that it links the ideas of the absorbing characteristics with the impedance characteristics of materials. Thus, the factor k used in the determination of the normal absorption coefficient is used in the calculation of the resistive and reactive components of impedance of the material.

The experiment does not indicate a method for separating the two components of reactance, namely, the spring and the mass factors. Information on the magnitudes of these two components may be estimated by making certain simplifying assumptions especially if the absorbing material has a marked absorption maximum. In diaphramatic materials,[6] this absorption maximum may be related to a minimum reactance. By equating the mass reactance to the spring factor reactance, the frequency for maximum absorption may be approximated.

Absorption may be expected to be high[7] when the acoustical impedance is equal approximately to the impedance of the air. Material may therefore be made up so as to have reactances (X) of zero magnitude, or near zero magnitude at the frequency desired for maximum absorption. The dissipative (R) factor may also be varied in a manner to approach the value of that for air. Usually, the problem is to design acoustical absorption material so as to obtain a reasonably constant impedance over a wide frequency range in order to produce a reasonably uniform absorption coefficient.

[6] L. B. Ham and H. T. Darracott, "Vibratory characteristics of vibraframe," *J. Acous. Soc. Am.* **11**, 333–340 (1940).

[7] H. J. Sabine, "Sound absorption and impedance of acoustical materials," *J. Motion Picture Engrs.* **49**, No. 3, 262–278, (1947).

It is when the pirates count their booty that they become mere thieves.—WILLIAM BOLITHO.

Teaching Alternating Current Circuits

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IT is likely that teachers of the first-year general course in physics would hesitate to give to their students the problem of solving an alternating current circuit similar to that shown in Fig. 1. This type of circuit would ordinarily be one for advanced students in electrical engineering. The equivalent direct current circuit, however, would not be considered too difficult for students of elementary physics. The purpose of this article is to describe a method for solution of some kinds of steady state a.c. circuit problems which is, in principle, no more difficult than the solution of equivalent direct current circuit problems.

For the d.c. circuit equivalent to Fig. 1, the straightforward application of Kirchhoff's laws results in eight simultaneous equations not all independent to be solved for a number of unknown quantities. The solution of these equations represents much more of a mathematical effort than is usually required of first-year physics students and the method of solving such circuits by direct application of Kirchhoff's laws is usually avoided. The method of simplification of circuit is the one most easily learned. This method can be applied to a.c. circuit problems in nearly the same manner as for d.c. circuit problems. The only difference between the two solutions is that for a.c. circuit problems additions are vector rather than scalar.

To determine the effective potential drop, effective current, and power delivered between any two points of the circuit in Fig. 1, the circuit is first represented as made up of impedances. Impedance $Z=V/I$, where V is the effective potential difference between two points and I is the effective current between those two points. Impedances are inductive, resistive, capacitive, and combinations of these. The relation between inductive impedance and inductance is $Z=2\pi fL$, where L is the inductance and f is the frequency; that between resistive impedance and resistance is $Z=R$; and that between capacitive impedance and capacitance is $Z=1/2\pi fC$, where C is the capacitance.

The laws which describe alternating current circuits may be considered as laws discovered by experiment. Suppose the potential drops in the series circuit of Fig. 2 are measured with a voltmeter which reads effective potential differences. It is discovered that with switch K closed $V_{AD}\neq V_{AB}+V_{BC}+V_{CD}$ where V_{AD}, V_{AB}, V_{BC}, V_{CD} represent voltmeter readings. When, however, the voltages are added as vectors, as shown in Fig. 3, it is found that $V_{AD}=V_{AB}+V_{BC}+V_{CD}$. In taking the vector sum of potential drops, if the potential drop across inductance is considered as a vector vertically upward, the potential drop across resistance is a vector horizontally to the right, and the potential drop across capacitance is a vector vertically downward. Further measurements of electromotive force between D and G, G and A and D and A, Fig. 2, made with switch K open so that no current flows through the generators, shows that $E_{AD}\neq E_{AG}+E_{GD}$. If, however, the electromotive forces are considered as vectors such that the angle between E_{AG} and E_{GD} is the phase difference $(\phi_2-\phi_1)$ between the two generators it is found that $E_{AD}=E_{AG}+E_{GD}$ or $E=E_1+E_2$, as shown in Fig. 4. Any convenient direction is chosen for E_1, and E_2 is drawn in a direction making an angle $(\phi_2-\phi_1)$ with E_1.

These measurements and others like them are

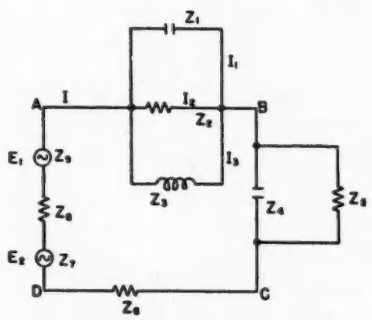

FIG. 1. Alternating current circuit.

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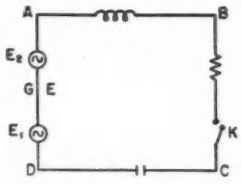

FIG. 2. Series alternating current circuit.

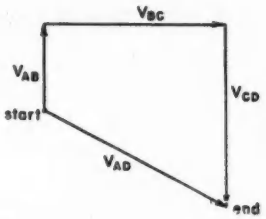

FIG. 3. Vector addition of voltages for series a.c. circuit.

described by Kirchhoff's first law for alternating current circuits: (1) *In any closed path the vector sum of the potential drops equals the vector sum of the electromotive forces.*

With an ammeter which reads effective current, measurements of the total current through a group of impedances in parallel and the separate currents through each of the individual impedances, as for example in the circuit of Fig. 1, show that $I \neq I_1+I_2+I_3$. If, however, current is considered as a vector, then $I=I_1+I_2+I_3$ as shown in Fig. 5. In making the vector sum, if current through inductance is considered as a vector upwards, current through resistance is a vector horizontally to the right, and current through capacitance is a vector vertically down. Kirchhoff's second law for alternating current circuits may therefore be stated: (2) *At any point in a network the vector sum of the currents approaching that point equals the vector sum of the currents leaving that point.*

For impedances in series, from Kirchhoff's first law for alternating current circuits one may write, for any two points in the circuit, $V=V_1+V_2+V_3+\cdots$ where V is the potential difference between the two points and V_1, V_2, $V_3\cdots$ are the separate potential differences for impedances or groups of impedances in series between the two points. Replacing each potential difference by ZI and accepting as an experimental observation the fact that the current is the same for all impedances in series, one obtains $ZI=Z_1I+Z_2I+Z_3I$. Cancelling I leaves $Z=Z_1+Z_2+Z_3+\cdots$. This is the same as the equation for resistances in series. For impedances, however, the equation represents a vector addition. If inductive impedance is considered as a vector in a direction vertically up, it is horizontally to the right if resistive, and vertically down if capacitive. The resultant impedance may be obtained by graphical addition.

For impedances in parallel, the second of Kirchhoff's laws for alternating current circuits may be written $I=I_1+I_2+I_3$, where I_1, I_2, and I_3 are vectors to be added graphically. Experiment shows that the potential difference across impedances in parallel is the same for all impedances. Since $I=V/Z$, then $(V/Z)=(V/Z_1)+(V/Z_2)+(V/Z_3)$. Cancelling V leaves $(1/Z)=(1/Z_1)+(1/Z_2)+(1/Z_3)$ where again one finds that the equation for impedances in parallel is the same as that for resistances in parallel except that for impedances in parallel the addition must be made by the vector method. In making this addition if $1/Z$ is considered as a vector vertically up for an inductive impedance, $1/Z$ is horizontally to the right for a resistive impedance and vertically down for a capacitive impedance.

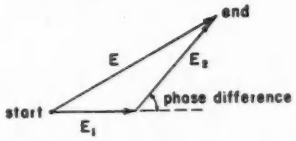

FIG. 4. Vector addition of electromotive forces for series a.c. circuit.

The circuit of Fig. 1 can now be solved by the method of simplification of circuit following the same procedure as that followed for direct current circuits except for the substitution of vector addition for scalar addition. Each group of impedances in parallel is first reduced to a single impedance using the equation $(1/Z_0)=(1/Z_1)+(1/Z_2)+(1/Z_3)$. The vector addition of these reciprocals is shown in Fig. 6 and the resultant vector, $1/Z_0$ shows the direction to be ascribed to Z_0, the impedance of the group. Repeating the procedure for the second group in parallel, the vector addition is shown in Fig. 7. The resultant impedance Z_{10} and its direction are thus obtained. The circuit has now been reduced to one with a number of impedances all in series, as shown in Fig. 8. With application of the equation for impedances in series $Z=Z_1+Z_2+\cdots$, one obtains the resultant impedance from the vector addition shown in Fig. 9. The internal impedances of the

generators Z_7 and Z_9 are considered inductive in this problem. The resultant impedance Z and its direction are determined from measurements on the diagram. The two alternating current generators are replaced by a single alternating current generator by the vector addition of the electromotive forces, as shown in Fig. 4. The circuit is thus simplified to a single a.c. generator and single impedance as in Fig. 10. For this simplified circuit the current $I=E/Z$. This effective current for the simplified circuit is also the current for the series circuit, Fig. 8. From the current and individual impedances in the series circuit, all of the potential drops can be determined with the equation $V=ZI$. These potential drops are recognized to be the same as the potential drops between the corresponding points of the original circuit, Fig. 1. Since now all potential drops are known for the original circuit, the individual currents between any two points can be determined by the equation $I=V/Z$.

As a check on the accuracy of the results, the individual currents for several impedances in

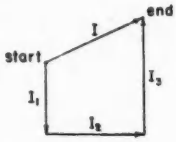

FIG. 5. Vector addition of currents for a group of impedances in parallel.

parallel may be added vectorially as in Fig. 5. The vector sum should equal in magnitude the current calculated for the simplified circuit.

The power delivered by the generators is $P=EIk$, where k is the power factor. The power factor for this circuit is Z_R/Z where Z_R is the horizontal component of the resultant impedance Z, as shown in Fig. 9.

Power expended is given by $P=EIZ_R/Z$. From Ohm's law, however, $E/Z=I$ where Z is the resultant impedance for the series circuit. Thus $P=Z_RI^2$. Here Z_R is the horizontal component of the entire impedance in the series

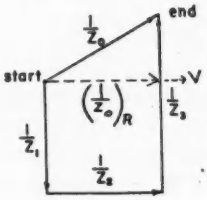

FIG. 6. Vector addition of reciprocal impedances in parallel group consisting of Z_1, Z_2, and Z_3.

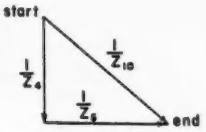

FIG. 7. Vector addition of reciprocal impedances for parallel group consisting of Z_4 and Z_8.

circuit, I is the current and P the power delivered by the generators.

The power delivered between any two points such as A and B in the series circuit, Fig. 8, is also $P=VI\times$power factor, where V is the voltage between the two points. The power factor A to B is $(Z_0)_R/Z_0$, in which $(Z_0)_R$ is the horizontal component of Z_0, as shown in Fig. 9.

Thus

$$P_{AB}=V_{AB}I\frac{(Z_0)_R}{Z_0}.$$

But by Ohm s law $V_{AB}/Z_0=I$, so $P_{AB}=(Z_0)_RI^2$.

In general, the power delivered between any two points in a series circuit is $P=Z_RI^2$, where Z_R is the horizontal component of the impedance between the two points.

The power delivered to any one impedance Z in a group of impedances connected in parallel is $P=VIk$, where in this case the power factor is given by $(1/Z)_R/(1/Z)$, where $(1/Z)_R$ is the horizontal component of $1/Z$, V is the potential difference across Z, and I is the current through Z. By Ohm's law $I=V/Z$. Thus $P=(V^2/Z)[(1/Z)_R/1/Z]$. The quantity $1/Z$ cancels and $P=V^2(1/Z)_R$, where P is the power delivered to any one impedance Z of a group in parallel and V is the potential drop across the group.

For either series or parallel connections the power is zero for pure inductive or capacitive impedances.

When all currents, potential drops, and powers are known, the a.c. circuit may be considered to be as completely solved as the equivalent d.c. circuit. For a.c. circuits, however, it is important to determine also the phase relations.

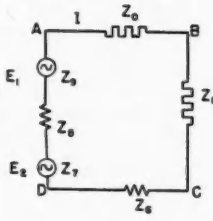

FIG. 8. Reduced circuit equivalent to circuit of Fig. 1.

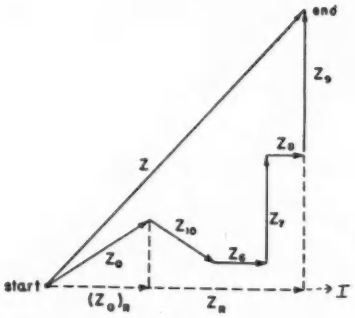

FIG. 9. Vector addition of impedances for series a.c. circuit of Fig. 8.

The phase relation between current and voltage for any part of a series circuit such as that in Fig. 8 is shown in Fig. 9. The current for any part of the series circuit is represented by a vector in the direction of Z_R; and the voltage between any two points, as a vector in the direction of the impedance between the two points. For the circuit as a whole the phase difference between current and voltage equals the angle between Z_R and Z. For the impedance Z_{10}, the phase difference between voltage and current is the angle between Z_R and Z_{10}.

To show lead and lag, vectors in Fig. 9 for parts of a series circuit may be considered as all rotating counterclockwise. For the given circuit as a whole the voltage is ahead of the current. For Z_{10}, the voltage is behind the current.

The phase relation between voltage and current for any one impedance Z in a group of impedances in parallel is shown by Fig. 6. The voltage for all impedances in the group is represented by a horizontal line parallel to $(1/Z_0)_R$ and the current through any impedance is considered to have the same direction as $1/Z$. Here Z_0 is the combined impedance of the group and $(1/Z_0)_R$ is the horizontal component of $1/Z_0$. The angle between the vector $1/Z$ and $(1/Z_0)_R$ is the phase difference between the current and voltage for the impedance Z.

The lead and lag are shown if, for impedances in parallel, the voltage, current, and $1/Z$ are considered to be vectors all rotating clockwise. Thus for the group of combined impedance Z_0, the voltage leads the current, but in the capacitive impedance Z_1, the current leads the voltage.

In the use of vector additions to determine resultant impedances for several impedances in series, or in parallel, it is recognized that impedance is not an ordinary vector. Voltage and current are ordinary vectors but sometimes called phasors, and impedance is a multiplier such that $V=ZI$. If I is not parallel to V, the impedance is such a multiplier that ZI is a vector parallel to V. When several impedances are connected in series, the resultant impedance is obtained by the vector addition of the separate impedances. What are really being added are the potential differences, not the impedances. The equation $Z=Z_1+Z_2+\cdots$ was obtained from $ZI=Z_1I+Z_2I+\cdots$ by cancelling I. The current has the same magnitude and direction for all, but Z_1I, Z_2I, and Z_3I are vectors in directions different from I. The magnitude of the current is cancelled. There remains a unit current vector multiplier for each impedance, so that each impedance times its unit current vector is a vector in the direction of the voltage across the impedance. It is convenient to take this into account by treating

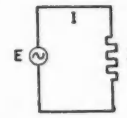

FIG. 10. Simplified series a.c. circuit equivalent to circuit of Fig. 1.

impedance as though it were a vector and thus obtain the resultant impedance for the group in series.

The same situation occurs when impedances in parallel are combined. The vector addition of $(1/Z)=(1/Z_1)+(1/Z_2)$ is really an addition of currents $(V/Z)=(V/Z_1)+(V/Z_2)\cdots$ with Z as a divider for the potential differences. The potential differences all have the same magnitude and direction. The impedance is of such a character that V/Z_1, V/Z_2, $\cdots$ are vectors in the directions of the currents. The magnitudes of the potential differences are cancelled. The directions, however, still persist so that $(1/Z)=(1/Z_1)+(1/Z_2)$ represents unit potential difference vectors divided by Z, Z_1, or Z_2. This is taken into account by considering that $1/Z$ for an impedance is a vector in the same direction as the current through that impedance.

In summary, the solution of an a.c. circuit problem like that of Fig. 1 consists of representing impedance, and the reciprocal of impedance, as vector quantities having directions vertically up if pure inductive, horizontally to the right if pure resistive, and vertically down if pure capacitive. With this convention, the procedure of simplification of circuit to a single electromotive force and impedance, followed by expansion back to the original circuit can be followed for both a.c. and d.c. circuits. This procedure enables a student of elementary physics to learn how to solve a.c. circuit problems with a minimum of difficulty. Those who go on to courses in electrical engineering will likely have little difficulty in learning a new convention for directions.

The authors wish to express thanks to Professor R. R. Benedict and Dr. Stanley C. Snowdon for helpful discussions.

The Laplace Transform as a Form of Curve Fitting

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THE importance of the Laplace transformation technique for the analysis of linear systems needs no justification in the minds of physicists and engineers. The ever increasing volume of literature appearing in scientific journals and employing these methods is sufficient motivation for teaching the fundamentals of the subject to physics students at an intermediate level.

The conventional presentation of the subject begins with the formal definition of the Laplace transform of a function of time; that is, with the formula,

$$\mathfrak{L}f(t) \equiv \int_0^\infty f(t)e^{-st}dt \equiv F(s). \qquad (1)$$

It is pointed out that this transformation, in general, converts an integro-differential equation in real variables to an algebraic equation in complex variables. The resulting simplification in mathematical manipulation is obvious to all. Now the more sophisticated students are able to accept this presentation without reservation. The majority, however, find great difficulty in developing an early confidence in the method since they see no reason why this definition was adopted other than, "it works." In short, they have no physical insight into the process and have no mental picture to accompany their manipulations. The situation becomes even more confused after the introduction of the inverse transformation formula,

$$f(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)e^{st}ds. \qquad (2)$$

This relation is presented supposedly to complete the theory of the process but is seldom used in physical problems. The unusual limits are confusing, to say the least.

The author has found that the basic Laplace formulas can be developed in a few easy stages from the well-known process of curve fitting and that a knowledge of this development leads to clearer interpretations of the Laplace techniques when applied to physical problems. It seems appropriate, therefore, to present such a treatment of the Laplace transformations to the readers of the *Journal*. The presentation will begin with a brief review of the curve fitting process.

Elements of Curve Fitting

Suppose there is given a function of time such as curve A of Fig. 1. It is well known that an assumed arbitrary function can be made to pass through n specified points of the given curve provided there are n constants in the assumed function that can be adjusted. For example, the function $f(t)(=)a_0+a_1t$, in which the symbol $(=)$ is used to indicate equality at a finite number of points, can be made to pass through the points $f(t_1)$ and $f(t_4)$ as indicated by the curve B since there are two adjustable constants, a_0 and a_1. On the other hand, the function $f(t)(=)a_0+a_1t+a_2t^2+a_3t^3$ can be made to pass through the four points, $f(t_1)$, $f(t_2)$, $f(t_3)$, and $f(t_4)$ as shown in the curve C. The improvement in the accuracy of the fitting process comes from an increase in the number of points of intersection. If this idea is extended, an exact fit can be obtained provided the assumed function agrees with the given curve at all points. This implies that there must be an infinite number of constants which can be adjusted.

It is important to note that the form of the assumed function is not unique. For example, the curve of Fig. 1 could be fitted at the same four points by the function $f(t)(=)a_0+a_1t+a_2t^2$

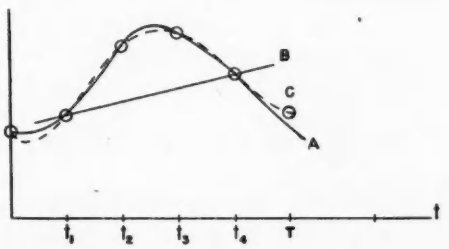

FIG. 1. Curves approximating a given function.

$+a_3 \sin 2t$. Hence, the choice of the fitting function must be dictated by other considerations, chief of which is ease in evaluating the constants in the assumed expression. Such a choice will now be made which is a series expression and which includes the Fourier series as a special case.

A General Expression for Fitting a Curve

A simple, yet versatile form for fitting a curve exactly in the interval 0 to T is the series expression,

$$f(t)=\frac{2}{T}\left[\frac{a_0}{2}e^{ct}+a_1e^{ct}\cos\frac{2\pi}{T}t\cdots a_ne^{ct}\cos\frac{2\pi n}{T}t\cdots +b_1e^{ct}\sin\frac{2\pi}{T}t\cdots b_ne^{ct}\sin\frac{2\pi n}{T}t\right], \quad (3)$$

where c is an arbitrary constant. This series can be written more compactly as

$$f(t)=\frac{2}{T}e^{ct}\left[\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{2\pi n}{T}t+\sum_{n=1}^{\infty}b_n\sin\frac{2\pi n}{T}t\right]. \quad (4)$$

It will now be shown that the constants, a_n and b_n, can be computed from simple formulas and that the basic Laplace transformations (Eqs. (1) and (2)) follow from this expression for curve fitting.

If Eq. (4) is multiplied by $e^{-ct}dt$ and integrated from 0 to T, it follows that,

$$a_0=\int_0^T f(t)e^{-ct}dt. \quad (5)$$

Likewise, by multiplying by $e^{-ct}\cos(2\pi n/T)tdt$ and $e^{-ct}\sin(2\pi n/T)tdt$ in turn and integrating, the two relations,

$$a_n=\int_0^T f(t)e^{-ct}\cos(2\pi n/T)tdt, \quad (6)$$

$$b_n=\int_0^T f(t)e^{-ct}\sin(2\pi n/T)tdt, \quad (7)$$

are obtained.

These last three equations can be made more compact. Let $2\pi n/T=\omega_0$ and let j represent the imaginary number $(-1)^{\frac{1}{2}}$. An easy calculation will show that

$$a_n-jb_n=\int_0^T f(t)e^{-ct}e^{-jn\omega_0t}dt\equiv F(c, n\omega_0), \quad (8)$$

of which c is an assumed constant. This is the basic formula for computing the constants of Eq. (4) and is true for any value of n including zero. A similar calculation shows that

$$a_n+jb_n=\int_0^T f(t)e^{-ct}e^{jn\omega_0t}dt\equiv\bar{F}(c, n\omega_0). \quad (9)$$

It is to be noted that $\bar{F}(c, n\omega_0)$ is the complex conjugate of $F(c, n\omega_0)$ and that

$$\bar{F}(c, n\omega_0)=F(c, -n\omega_0), \quad (10)$$

$$a_n=\tfrac{1}{2}[\bar{F}(c, n\omega_0)+F(c, n\omega_0)], \quad (11)$$

and

$$b_n=\frac{1}{2j}[\bar{F}(c, n\omega_0)-F(c, n\omega_0)]. \quad (12)$$

If the basic curve fitting relation (4) is reconstructed in the light of Eqs. (11) and (12), it is found that

$$f(t)=\frac{\omega_0}{2\pi}e^{ct}[F(c, 0)+\sum_{n=1}^{\infty}F(c, n\omega_0)e^{jn\omega_0t}+\sum_{n=1}^{\infty}\bar{F}(c, n\omega_0)e^{-jn\omega_0t}]. \quad (13)$$

By Eq. (9), this becomes

$$f(t)=\frac{\omega_0}{2\pi}e^{ct}[F(c, 0)+\sum_{n=1}^{\infty}F(c, n\omega_0)e^{jn\omega_0t}+\sum_{n=1}^{\infty}F(c, -n\omega_0)e^{-jn\omega_0t}].$$

Let n be replaced by $-n$ in the last summation, yielding

$$f(t)=\frac{\omega_0}{2\pi}e^{ct}[F(c, 0)+\sum_{n=1}^{\infty}F(c, n\omega_0)e^{jn\omega_0t}+\sum_{n=-1}^{-\infty}F(c, n\omega_0)e^{jn\omega_0t}].$$

It is obvious that the right-hand side of the last equation can be represented by the single sum-

mation

$$f(t)=\frac{\omega_0}{2\pi}e^{ct}\sum_{n=-\infty}^{\infty}F(c, n\omega_0)e^{jn\omega_0 t}. \qquad (14)$$

Let us recapitulate the steps that have been taken. We have adopted the expression

$$\frac{2}{T}e^{ct}\left[\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{2\pi n}{T}t+\sum_{n=1}^{\infty}b_n\sin\frac{2\pi n}{T}t\right]$$

as particularly suitable for fitting exactly a function of time between the limits $t=0$ and $t=T$. The expression involves an arbitrary constant c, the need for which will appear later, and a set of constants, a_n and b_n. It was shown that any pair of constants a_n and b_n can be calculated from the relation

$$a_n-jb_n=\int_0^T f(t)e^{-ct}e^{-jn\omega_0 t}dt=F(c, n\omega_0). \qquad (15)$$

It was also shown that if the constants are known, then the expression can be constructed by means of the relation

$$f(t)=\frac{\omega_0}{2\pi}e^{ct}\sum_{n=-\infty}^{\infty}F(c, n\omega_0)e^{jn\omega_0 t}. \qquad (16)$$

These relations (15) and (16) can be viewed as direct and inverse transformations, the first generating a set of coefficients when a function is known, the second generating a function when the coefficients are given. Doubtless, the mathematically inclined will be bothered at this stage with existence considerations. These have been passed over roughshod in an effort to establish a simple sequence of ideas. It should be obvious, however, that theorems for the existence of Fourier series apply now to the function $e^{-ct}f(t)$ rather than $f(t)$. This leads to greater freedom as will be mentioned later.

Extension of the Range of Representation

So far, the given function $f(t)$ has been fitted only over the finite time interval 0 to T. Suppose the interval is enlarged until T approaches infinity? Let us assume this can be done. It is to be observed that $\omega_0=(2\pi/T)$ approaches zero. Now ω_0 is the common difference in angular frequency of the harmonic factors between consecutive values of n. Hence, in the limit, $F(c, n\omega_0)$ or a_n and b_n exist at all frequencies. Let us simplify our expression by writing ω for $n\omega_0$. The expression (15) becomes in the limit

$$F(c, \omega)=\int_0^{\infty}f(t)e^{-ct}e^{-j\omega t}dt. \qquad (17)$$

Two remarks are in order. In the first place, even though a_n and b_n, as defined by Eq. (4), may be finite, the coefficient of each term in the basic series is an infinitesimal since each term is multiplied by $2/T=\omega_0/\pi$. In the second place, Eq. (17) is just like a Fourier transformation except that $f(t)e^{-ct}$ replaces the usual $f(t)$ as found in the Fourier integral. Hence our processes are valid for the class of functions $f(t)e^{-ct}$ for which a Fourier integral representation is possible; hence any existence theorem on such integrals can be applied directly to our situation. Let it be recalled that c is an arbitrary constant. It is just this freedom in the choice of c which allows us to make some functions well-behaved which would not be otherwise. For example, the common excitation function,

$$\begin{aligned} f(t)&=0, \quad t<0\\ &=A\sin pt, \quad t>0, \end{aligned}$$

has no Fourier representation but can be represented by our relation (4) provided $c>0$.

The role played by the arbitrary constant c can be sensed in a general way by examining our curve fitting process from another point of view. Equation (4) shows that $f(t)$ is fitted exactly by an infinite set of sinusoids which may decay, expand, or be of constant amplitude depending on the value of c. Consider fitting the function, $f(t)=Ae^{-pt}$. Equation (17) can exist only if $c>-p$, that is, if the individual terms in Eq. (4) decay more slowly that the original function. Again, if $f(t)=Be^{kt}$, then $c>k$; i.e., the terms of Eq. (4) expand at a faster rate than e^{kt}. As a third example, consider the common step-function

$$\begin{aligned} f(t)&=0, \quad t<0\\ f(t)&=1, \quad t>0. \end{aligned}$$

This function has no Fourier representation but can be fitted by Eq. (4) provided $c>0$. Since the

original function neither decays nor expands, it can be fitted provided our terms expand at any finite rate. Similar considerations will show that our basic relation cannot be made to fit the function $f(t)=D(e^{t^2})$, since no finite value of c can be found which will make the terms in our relation expand at a faster rate than the original function. Curve fitting with harmonic terms of constant amplitude is well known from the Fourier series but fitting by use of expanding terms is not so well known even though they are more versatile.

If T approaches infinity, Eq. (16) becomes

$$f(t)=\frac{1}{2\pi}\sum_{\omega=-\infty}^{\infty}e^{ct}F(c,\omega)e^{j\omega t}\omega_0.$$

But ω_0 becomes infinitesimally small in the limit; hence the sum can be represented by the integral,

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{ct}F(c,\omega)e^{j\omega t}d\omega. \tag{18}$$

Equations (17) and (18) can be called direct and inverse transformation formulas in the same fashion as for Eqs. (15) and (16); useful when the range of representation extends from $t=0$ to $t=\infty$.

The Laplace Transformation Formulas

An examination of the relations (14)–(18) discloses that c and ω always occur in the combination $c+j\omega$. Therefore, let a new variable s be defined by the relation

$$s=c+j\omega. \tag{19}$$

It follows that $ds=jd\omega$, $s=c-j\infty$ when $\omega=-\infty$, and $s=c+j\infty$ when $\omega=+\infty$. Using these relations, Eqs. (17) and (18) become

$$F(s)=\int_0^{\infty}f(t)e^{-st}dt, \tag{20}$$

and

$$f(t)=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}F(s)e^{st}ds, \tag{21}$$

which are the direct and inverse Laplace transformation formulas.

Summary

It is well known that there are numerous sets of functions which can be employed to approximate a more or less arbitrary function to any desired accuracy over a finite real interval. If then a set is chosen, each term of which is an exponentially decaying or expanding sinusoid, the sum of such a set can be made to fit a broad class of physical functions. Formulas for computing the coefficients of the terms (direct transformation) and for generating the function from a set of the coefficients (inverse transformation) can be established. These formulas are simple and compact. When the range of representation is extended from $t=0$ to $t=\infty$, the formulas still remain simple and compact and are valid for most physical functions by virtue of an arbitrary constant which determines the rate of decay or growth of the individual terms in the series representation. The Laplace transformation formulas follow naturally.

There is another point which is important with regard to this country's position in particular. If we adopt policies that are too restrictive about scientific knowledge, other countries will also do the same. As CHARLES KETTERING *of the General Motors Corporation has put it, "When you lock the doors of the laboratory, you lock out more than you lock in." This result could be especially weakening to America for we have not thus far been very productive in truly fundamental science. On the other hand we are vevy well equipped to make rapid progress in exploiting technological uses of such fundamental knowledge. For these reasons we have to guard against policies that are so narrowly restrictive that we stifle our own development.*—E. U. CONDON (1948).

The Action Principles

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STUDENTS of the foundations of mechanics, particularly Heinrich Hertz,[1] Ernst Mach,[2] and Karl Pearson,[3] have criticized Newton's laws of motion and have pointed out their limitations. That these laws do not form a sufficiently satisfactory foundation of mechanics is brought out by the fact that, for about sixty years after the publication of Newton's *Principia*, eminent physicists and mathematicians had to have recourse to other laws in order to solve many problems of mechanics.

> The postulate of Huyghens, "that if any weights are put in motion by the force of gravity they cannot move so that the center of gravity of them all shall rise higher than the place from which it descended," was generally one of the principles of the solution: but other principles were always needed in addition to this, and it required the exercise of ingenuity and skill to detect the most suitable in each case. Such problems were for some time a sort of trial of strength among mathematicians. The *Traité de Dynamique* published by D'Alembert in 1743 put an end to this kind of challenge by supplying a direct and general method of resolving, or at least throwing into equations, any imaginable problem. The mechanical difficulties were in this way reduced to difficulties of pure mathematics.[4]

The limitations of Newton's laws are due to the facts that the first law is a special case of the second; the second law applies directly to a body which may be considered as a particle; and the third law applies to two interacting particles. It must be stated that laws applicable to more complex systems can be derived from Newton's laws; but the presentation of mechanics becomes simpler when the subject is developed from a broad general principle from which laws applicable to special cases are derived, than when this process is reversed. For this reason, the presentation of advanced mechanics is far more coherent than that of elementary mechanics.

Since the publication of D'Alembert's principle, a number of other general principles have been formulated and made the basis of advanced texts on mechanics. Unfortunately, none of these principles is suited to elementary mechanics. Even the simplest of them, D'Alembert's principle, is too formidable to be placed in the hands of the beginner in its original form. This principle can, however, be stated and is stated, in my *Analytical Mechanics*,[5] in a form which is suited to elementary texts.

The new form consists of twin principles which I have called the *linear action principle* and the *angular action principle*. The linear action principle states:

> The vector sum of all the external linear actions to which a system of particles or any part of it is subject at any instant equals zero. $\sum \mathbf{A} = 0$.

The angular action principle, which is derived from the linear action principle, states:

> The vector sum of all the external angular actions to which a system of particles or any part of it is subject at any instant equals zero. $\sum \mathbf{A}_a = 0$.

Linear actions are divided into two classes, namely, *forces* and *kinetic reactions*. A force is a vector which represents the pull or push of one particle on another. A kinetic reaction is a vector which represents the action of the material universe (the *inertial field*) on a particle. It acts only while the particle is accelerated. It is proportional to and opposed to the acceleration. The constant of proportionality is the *mass* of the particle. With these definitions the linear

[1] Heinrich Hertz, *Principles of mechanics* (Macmillan).
[2] Ernst Mach, *Science of mechanics* (Open Court, ed. 5, 1942).
[3] Karl Pearson, *Grammar of science* (A. & C. Black, London, ed. 2, 1900).
[4] Edward J. Routh, *Elementary rigid dynamics* (Macmillan, ed. 7, 1905).
[5] H. M. Dadourian, *Analytical mechanics*, (Van Nostrand, ed. 3, 1931).

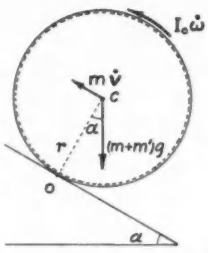

FIG. 1. Motion of a water-filled spherical shell on an inclined plane. Rotation is considered about the point of contact between sphere and plane.

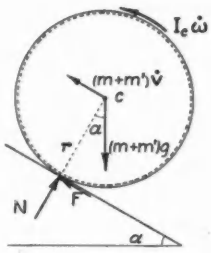

FIG. 2. Motion of a water-filled spherical shell on an inclined plane. Rotation is considered about the center of the shell.

action principle may be written in the form,

$$\sum(\mathbf{F}-m\dot{\mathbf{v}})=0, \tag{1}$$

where $\mathbf{F}$ is the force acting on a particle of mass m that possesses a velocity v.

Angular actions are divided into two classes, namely, *torques* and *angular kinetic reactions*. A torque is the moment of a force; and an angular kinetic reaction is the moment of a (linear) kinetic reaction. Hence the angular action principle may be written in the form

$$\sum(\mathbf{G}-m\dot{\mathbf{v}}\mathbf{p})=0, \tag{2}$$

where $\mathbf{G}$ denotes torques and $\mathbf{p}$ the lever arm of $m\dot{\mathbf{v}}$. (The vector product sign is omitted in Eq. (2) for pedagogical reasons and also because rotational motions considered in elementary mechanics are uniplanar motions, in which case the vector product notation need not be introduced.)

When resolved into component equations, Eq. (1) yields the first three, and Eq. (2) the last three of the six differential equations of D'Alembert's principle.

If the system consists of a single particle, or many particles, each of which has the same acceleration, Eq. (1) reduces to

$$\sum\mathbf{F}-m\dot{\mathbf{v}}=0, \tag{1'}$$

which is Newton's second law of motion.

If the system consists of a rigid body in uniplanar motion, Eq. (2) reduces to

$$\sum\mathbf{G}-I\dot{\omega}=0, \tag{2'}$$

when I remains constant during the motion; and to

$$\sum\mathbf{G}-d/dt(I\omega)=0, \tag{2''}$$

when I varies.

The conditions for the equilibrium of a particle, Newton's third law, the law on the sum of internal forces, the law of the motion of the center of mass, the principle of the conservation of linear momentum, etc., can be derived from Eq. (1). Analogous conditions, laws and principles which relate to rotation can be derived from Eq. (2).

Advantages of the Action Principles

The action principles, in spite of their broad generality, can be understood by the student, to the extent of his needs, at any stage of his study of mechanics. The principles lend themselves to a gradual unfolding of their implications and, consequently, to the presentation of mechanics in a logically coherent and gently graded way. The almost identical wording of the two principles brings out clearly the fact that there is a perfect analogy between the magnitudes involved in motion of translation and motion of rotation.

The concept of kinetic reaction affords the operational definition of kinetic energy as the result of the work done by a force against the kinetic reaction, as potential energy is defined as the result of the work done against a conservative force.

The wording of each principle brings out clearly certain points of great pedagogical importance. The action principle makes clear, for instance, the fact that internal forces should not be considered, and the fact that forces which act *on* (not those exerted *by*) the body whose equilibrium or motion is discussed, should be considered.

The clear-cut distinction between force and kinetic reaction eliminates the contradictions and inconsistencies which result from the use of the terms "fictitious force," "imaginary force," "centrifugal force," and "reversed effective force." The contradiction between the condition of equilibrium, according to which there is no acceleration when the sum of the forces equals zero, and the condition of motion, in which there is acceleration although the sum of the "forces"

equals zero is removed by the application of the action principles. The inconsistency of calling the normal component of the kinetic reaction *centrifugal* force without giving the tangential component a special name, say, *afugal* force (away-fleeing force) and the inconsistency of designating by the word *force* both the cause and the effect of acceleration cannot be avoided by the device of attaching an adjective to the word.

> Can we without confusing our ideas, suddenly begin to speak of forces which arise through motion, which are a consequence of motion? Can we behave as if we had already asserted anything about forces of this new kind in our laws, as if by calling them forces we could invest them with the properties of forces? These questions must be clearly answered in the negative. The only possible explanation is that, properly speaking, centrifugal force is not a force at all.[1]

Problems, such as the following, that would appear difficult to a student who has learned only Newton's laws, should be relatively simple to one who is trained in the application of the action principles.

Discuss the motion of a thin spherical shell, which rolls down an inclined plane; the shell is full of water and its inner surface is perfectly smooth.

Solution: (a) Taking the axis of rotation through the point of contact, and applying the angular action principle to Fig. 1, we get

$$(m+m')gr\sin\alpha - m\dot{v}r - I_0\dot{\omega} = 0,$$

or

$$(m+m')gr\sin\alpha - m\dot{v}r - 5m'r\dot{v}/3 = 0,$$

where m is the mass of the water and m' the mass of the shell, r is the radius of the shell, α is the angle of the plane to the horizontal, $\dot{v}$ is the linear acceleration of the center of mass, $\dot{\omega}$ is the angular acceleration of the spherical shell, and I_0 is the moment of inertia of the shell relative to the axis through point O, perpendicular to the plane of motion. Hence

$$\dot{v} = 3\frac{(m+m')\sin\alpha}{3m+5m'}g.$$

Since the acceleration is constant, we have $v = v_0 + \dot{v}t$, and $s = v_0t + \frac{1}{2}\dot{v}t^2$.

(b) Taking the axis of rotation through the center of the shell, and applying both action principles to Fig. 2, we have

$$(m+m')g\sin\alpha - F - (m+m')\dot{v} = 0,$$
$$N - (m+m')g\cos\alpha = 0,$$

and

$$Fr - I_c\dot{\omega} = 0,$$

where I_c is the moment of inertia relative to the axis through the center of the shell. Solving these equations for $\dot{v}$, we obtain the same result as that in Part (a).

Discussion: If $m=0$, that is, if the shell is empty, $\dot{v} = \frac{3}{5}g\sin\alpha$, as it should. If $m'=0$, that is, if the mass of the shell is negligible, $\dot{v} = g\sin\alpha$; hence the motion is the same as that of a body which slides down a smooth inclined plane.

He is still at play, save only that his play is such as manhood stops to watch, and his playthings are those which the gods gave their children. The universe is his box of toys. He dabbles his fingers in the day-fall. He is gold-dusty with trembling amidst the stars. He makes bright mischief with the moon. The meteors nuzzle their noses in his hand. He teases into growling the kennelled thunder, and laughs at the shaking of its fiery chain. He dances in and out of the gates of heaven; its floor is littered with his broken fancies. He runs wild over the fields of ether. He chases the rolling world. He gets between the feet of the horses of the sun. He stands in the lap of patient Nature, and twines her loosened tresses after a hundred wilful fashions, to see how she will look nicest in his song.—FRANCIS THOMPSON, describing SHELLEY. Quoted by A. S. EVE as applicable to LORD RUTHERFORD.

NOTES AND DISCUSSION

A Simple Fresnel Diffraction Experiment

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THE following laboratory experiment involving Fresnel diffraction has been devised to enable our students to study the phenomena of diffraction in the elementary laboratory and to measure the wavelength of sodium light with a fair degree of accuracy. The necessary equipment is available in any elementary optics laboratory and the experiment may be performed in a partially darkened room.

The essential parts of the apparatus are shown in Fig. 1. The point source of light L consists of a sodium lamp with a pinhole screen placed in front of it. The circular aperture D, greatly magnified in Fig. 1, is a 2-mm hole drilled in a sheet of thin metal and mounted on an optical bench. The observation of the diffraction pattern is accomplished by a simple lens or instrument eyepiece P mounted on the optical bench behind the aperture in such a way that the eyepiece may be moved along the bench. A set of cross hairs or an indicating pointer is mounted at the focal plane of the eyepiece lens to provide a reference point for the center of the diffraction pattern. As the eyepiece is moved away from the circular opening, fewer Fresnel zones of the circular aperture D contribute to the illumination of the center of the diffraction pattern. Therefore, the center of the

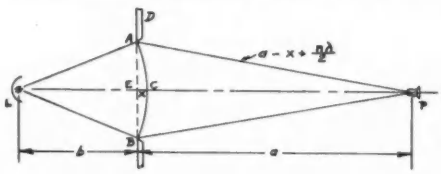

FIG. 1. Diagram showing the relationship between the wavelength of the light used and the constants of the apparatus.

pattern becomes alternately dark and bright as an even or odd number of zones contribute to the illumination.

In performing the experiment the student places the eyepiece near to the circular aperture D and slowly moves it away, noting the distances to the focal plane of the eyepiece from the circular aperture at the consecutive positions of maximum brightness and darkness of the center of the observed pattern. For a source distance of 2 meters and an optical bench of the same length, approximately 15 positions of light and dark centers can readily be identified.

The following theory will show how the data obtained may be used to determine the wavelength of the light used. As shown in Fig. 1, let a be the distance from observed pattern to circular opening, b be the distance from point source to circular aperture, r be the radius of circular aperture, n be the number of Fresnel zones contributing to a given diffraction pattern, x be the sagitta of the wave front arc ACB, and λ be the wavelength of light.

In the right triangle AEP

$$a^2+r^2=(a-x+n\lambda/2)^2 \qquad (1)$$

or

$$r^2=-2ax+an\lambda+x^2-xn\lambda+n^2\lambda^2/4. \qquad (2)$$

Since $a\gg x$ and $a\gg\lambda$, we may neglect the last three terms of Eq. (2) and obtain

$$r^2=-2ax+an\lambda. \qquad (3)$$

The approximate sagitta equation for the condition when $b\gg x$ is

$$x=r^2/2b. \qquad (4)$$

Eliminating x between Eqs. (3) and (4) we obtain

$$1/b+1/a=n\lambda/r^2. \qquad (5)$$

It is interesting to note that Eq. (5) has the same form as the usual lens or mirror equation. Equation (5) may be written in the following form

$$n[ab/(a+b)]=r^2/\lambda. \qquad (6)$$

Since the right hand member of Eq. (6) is a constant, the quantity $n[ab/(a+b)]$ is also a constant for the corresponding values of n and a.

There are several conditions imposed upon the series of whole numbers n which will help the student to correlate the different values of a with the appropriate n. For a light-centered diffraction pattern n is an even number and an odd number for a dark-centered pattern. Also, the values of n are consecutive whole numbers for consecutive light and dark centers, the smallest value of n corresponding to the greatest distance CP.

TABLE I. Typical data.*

Center of pattern	a (cm)	$\frac{ab}{a+b}$	Corresponding n	$n\left(\frac{ab}{a+b}\right)$
L	10.5	10.03	15	150.5
D	11.4	10.77	14	150.8
L	12.4	11.74	13	152.6
D	13.6	12.86	12	154.3
L	14.5	13.63	11	149.9
D	16.3	15.20	10	152.0
L	18.0	16.70	9	150.3
D	20.6	18.90	8	151.2
L	24.1	21.82	7	152.7
D	28.2	25.12	6	150.7
L	34.1	29.64	5	148.2
D	44.7	37.40	4	149.6
L	66.0	51.40	3	154.2
			Mean	151.3

* $b=232.0$ cm; $r=0.0947$ cm. Calculated wavelength 5920A. Wavelength of light used 5893A.

Thus by correlating each value of a with the appropriate n to obtain the best agreement with Eq. (6), a mean value of r^2/λ may be obtained. The diameter of the aperture having been determined by means of a measuring microscope, a value of the wavelength of the light may be obtained by the use of Eq. (6) and compared with the accepted value.

The data in Table I were chosen at random from student reports.

A Self-Sustaining Electronichord

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THE name "electronichord" was used by Urquhart[1] in describing a demonstration device for making audible the different modes of vibration of a plucked string. His apparatus consisted of a stretched wire whose ends were connected, through a matching transformer, to the input of an audio amplifier. Horizontal magnetic fields were applied to the wire by setting horseshoe magnets astraddle the wire. Plucking the wire in a vertical plane would then induce in it an alternating emf which was amplified and made audible by the amplifier and a loudspeaker.

With one magnet placed at the center of the wire, the audible tone would consist principally of the fundamental frequency of the vibrating string. Two magnets placed so that each was located at an antinode for the second harmonic, and oriented in opposite sense across the wire, would produce a tone consisting principally of the second harmonic. In similar fashion, any one of the modes of vibration of the plucked string might be made audible by placing the equivalent number of magnets across the wire in alternate orientation, each magnet being located at an antinode for the desired harmonic.

The writer has developed an elaboration of the original design and has called it the "self-sustaining electronichord." The modification consists of the addition of a transducer which is coupled mechanically to the wire and connected electrically to the output of the amplifier (see Fig. 1).

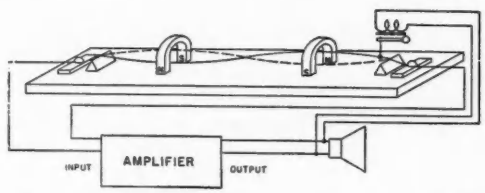

FIG. 1. Self-sustaining electronichord showing positions of magnets for vibration of the string in two loops.

Thus, when the wire is caused to vibrate in a vertical plane, the alternating emf induced in it is amplified and produces a similar vibration in the piston arm of the transducer. Since this arm is coupled to the wire, energy from the transducer is fed back into the wire. If this feedback vibration is in phase with the vibration of the wire, a sustained vibration of the string results.

The amplitude of the vibration is controlled by the setting of the gain control of the amplifier. The number of loops in which the string vibrates is determined by the number and the location of the magnets. For example, a single magnet placed at the center of the wire will cause it to vibrate in one loop, while two magnets placed at the antinodes for the second harmonic (as in Fig. 1) will cause vibration in two loops.

The transducer used by the author is the driving unit taken from a magnetic speaker. It is mounted above the stretched steel wire and mechanically coupled to it by a piece of No. 18 copper wire. The latter is soldered to the piston arm of the driver, carried down to the steel wire, wrapped around the steel wire and soldered. The location of the point of contact with the steel wire is fairly critical, as it determines the mechanical impedance matching between the vibrating string and the driving unit. In the author's apparatus it is located one-half inch from one end of the one-meter vibrating wire.

The amplifier normally used with this apparatus has two stages of voltage gain followed by an output stage which delivers up to 8 watts of power. With all of this amplification, it is not necessary to match the wire to the electrical input impedance of the amplifier. By matching both input and output electrical impedances properly, an amplifier consisting of one stage of voltage gain followed by a single 6F6 power output stage has operated the apparatus very nicely. The strength of the horseshoe magnets is not critical, and successful operation has been obtained with a wide variety of magnets found in the laboratory.

When the apparatus is connected for the first time, the gain control of the amplifier is turned full on and the string is gently plucked. If the transducer is connected in proper phase for positive feedback, the vibrations of the string will immediately build up. If this does not occur, it is usually found that the transducer has been connected in inverse phase. Its connections to the output of the amplifier are simply reversed, and the wire will then vibrate. Whenever the number and the location of the magnets are changed to cause the string to vibrate in some other harmonic mode, the magnet nearest the transducer must always have the same orientation; that is, its N pole must always be on the same side of the wire.

The self-sustaining feature has two advantages over the original electronichord. First, the selective action of positive feedback singles out the desired harmonic more effectively. When three magnets, for example, are placed at the antinodes for the third harmonic and the string is merely plucked, some of the other harmonics are heard along with the third. With the self-sustaining action, however, only the desired harmonic is produced. Second, the continuously sustained vibrations may be made with an amplitude so large that a class may see the string vibrate in addition to hearing the tone which it produces.

The author wishes to thank Mr. James W. Grooks for his help in building and testing this piece of equipment.

[1] N. Urquhart, *Am. J. Physics (Am. Physics Teacher)* **2**, 29 (1934).

Experiments in Static Electricity

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IN the discussion of electrostatic induction the introduction of the concept of potential adds materially to the effectiveness of the presentation. The simple idea of the separation of charges in a conductor under the influence of an electric field is inadequate. In the *Journal* for October 1940,[1] it was pointed out by the writer that a metal-pointed

rod, grounded, held near three charged balloons would have a potential of about −17,000 volts and that the resulting discharge from the point would neutralize the charge on the balloons causing them to fall together. Several additional experiments, which are best explained using the concept of potential, are described below.

1. If a pointed conducting rod is held by a handle made of a good insulator close to three balloons which have been charged by rubbing them with fur, no point discharge will take place. The potential difference between the rod and the balloons is much less than it is when the rod is grounded and the electric field at the point is not large enough to produce point discharge.

2. If the pointed end of the rod is now capped by a wooden cylinder, grounded, and again held near the balloons still no point discharge will take place. The potential difference between balloons and rod is the same as with an uncapped rod but the wooden cylinder prevents any repelled positive ions in the air from reaching the balloons and discharging the negative charge on them.

3. If the wooden cap is removed and the grounded rod held near the balloons, the latter are quickly discharged. If, however, a rod having many points at one end is grounded and held near, the point discharge will be very weak and the neutralization process very slow, since the field intensity at each point is much smaller than before.

4. If one rod is held by an insulating handle near the balloons and a second rod also held by an insulating handle is touched to the first and then separated, an induced charge appears on the first rod. The amount of the induced charge depends on the potential difference between the balloons and the rod. This may be measured by transferring some charge from the rod to an electroscope with a proof plane. If the experiment is repeated with the two rods grounded by touching one with the fingers, the induced charge will be found to be much greater than before. Using only the simple idea of the separation of charges by the inducing action of a charged object, the charge appearing on the rod in each of the above cases would be the same. Since, however, the potential difference is greater in the second case than in the first the induced charge appearing on the rod is correspondingly greater.

[1] Paul Rood, "An experiment in static electricity," *Am. J. Physics* 8, 320 (1940).

The Vibrating String Experiment

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ORDINARILY, students in our elementary physics courses do only three laboratory experiments in sound: *viz.*, the Siren Disk, Melde's Experiment, and the Kundt's Tube. This year, when seeking for an additional experiment, it was decided to try the Vibrating Strings Experiment using a sonometer. However, since much difficulty was encountered in attempting to tune the sonometer by the usual method of absence of beats, the following experimental arrangement and procedure were tried: A sonometer having three different types of strings was employed. The sounds produced by the strings were picked up by a crystal microphone, and converted to a variable electrical potential difference which was then amplified and applied to the horizontal deflecting plates of an oscilloscope. To the vertical deflecting plates of the oscilloscope there was applied a sinusoidal potential difference supplied by a Hewlett-Packard variable oscillator. The resulting Lissajous figures were then used to determine when the strings were properly tuned.

I. To investigate the dependence of frequency on length, the oscillator was first set to 128 vib/sec by holding a tuning fork of that frequency in front of the microphone and adjusting the oscillator until a stable pattern, an ellipse, resulted. Then one string, a meter long, was tuned to this same frequency by adjusting its tension. The oscillator was then set at a frequency of 256 vib/sec with the aid of a tuning fork. Using the same wire and tension, a movable bridge was adjusted until the wire's frequency became 256 vib/sec. The length of wire was measured with a meter stick. The same procedure was repeated for 512 vib/sec. Some typical data are given in Table I.

II. The dependence of frequency on tension was found by setting the tension on this same string at four pounds and adjusting the length with the movable bridge until the string was tuned to 128 vib/sec. At this constant length the tension on the string was increased until the string was tuned to 256 vib/sec. (In both cases the tuning was performed as previously described.) Table II gives some typical data.

TABLE I. Effect of length on the frequency of vibration of a stretched wire.

Length ratio	Reciprocal of length ratio	Frequency ratio	Percent difference
1.0	1.00	1.00	0
0.509	1.97	2.00	1.5
0.2505	3.99	4.00	0.5

TABLE II. Effect of tension on the frequency of vibration of a stretched wire.

Square root of tension	Square root of tension ratio	Frequency ratio	Percent difference
2.00	1.0	1.0	0
3.93	1.97	2.0	1.5

TABLE III. Effect of linear density on the frequency of a stretched wire.

Frequency	Square root of density ratio	Reciprocal of frequency ratio	Percent difference
256	1.0	1.0	0
127	2.03	2.02	0.5
121	2.16	2.12	1.8

III. To find the dependence of frequency on the linear density, when the length and tension were constant, each of the three strings had 16 pounds tension applied. The length of one was varied until the string was tuned to a frequency of 256 vib/sec. Then using this same length for each of the other two strings, the frequency of the oscillator was varied until a stable elliptical pattern resulted on the oscilloscope screen. The corresponding frequencies were read from the calibrated dial of the oscillator. Table III shows some typical data. As the data show, the accuracy is in the neighborhood of 1.5 percent, which is much better than could be obtained by the average student by the beat method of tuning.

With the apparatus used here, the chief error was in setting the spring balances at the end of the wires to the correct tension. A sonometer using weight pans and pulleys to supply the tension might possibly increase the accuracy of the results. However, taken as a whole, the experiment was very successful in teaching the laws of vibrating strings to our class in Liberal Arts Physics.

* Now at Ohio Wesleyan University.
** Now at Ohio State University.

Use of Motion Pictures in Laboratory Dynamical Studies

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IN student laboratory experiments in dynamics, as, for example, Newton's second law of motion or the conservation of momentum, it is, of course, highly desirable to allow motion with the least possible constraint and to observe the displacements of moving parts without using perturbing markers. This circumstance has led us to use motion pictures in the student laboratory for quantitative observation of rapid dynamic processes, a technique which is used in a well-known textbook.[1] The photographic method combines maximum freedom of movement with accurate observation of position as a function of time, if the camera frame speed is sufficiently high.

A double pendulum apparatus, with which we have studied the conservation of momentum in the student laboratory, is shown in Fig. 1. It consists of two wooden blocks, each suspended by a steel rod from a common free-swing pivot and counterweighted in such a way that the center of mass of each pendulum is at the center of its wooden block. A spring is compressed between the two blocks, and they are tied together with a thread. When this thread is burned, the pendulums recoil. The mass of each block can be varied at will.

The motion is photographed with a motion picture camera at 64 frames per second. The camera used in this work was a war-surplus gun-point aiming camera, adapted only to the extent of adding a viewfinder. Since there is a coordinate screen behind the pendulums, the student can determine the heights to which the blocks rise by examining the film with a magnifying viewer.

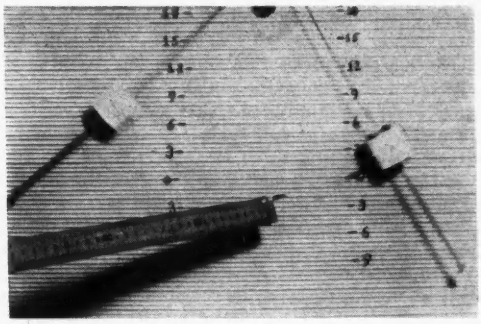

FIG. 1. A frame from movie film of the recoil pendulums flying apart after the burning of the binding thread. In this frame the pendulums are not near the end of their swing and are not completely stopped by the camera. The numbers are retouched for clarity in reproduction; they show clearly in the original.

Since each pendulum receives the same impulse, the initial angular momenta are equal, giving

$$(I_1+M_1r^2)\omega_1=(I_2+M_2r^2)\omega_2. \tag{1}$$

Here ω_1 and ω_2 are the initial angular velocities of the two pendulums, I_1 and I_2 are the respective moments of inertia about axes through the centers of mass parallel to the actual axis of rotation, M_1 and M_2 are the respective total masses, and r is the common distance from each center of mass to the axis of rotation. By the conservation of energy we have

$$\begin{aligned}\tfrac{1}{2}(I_1+M_1r^2)\omega_1^2&=M_1gh_1,\\ \tfrac{1}{2}(I_2+M_2r^2)\omega_2^2&=M_2gh_2,\end{aligned} \tag{2}$$

where h_1 and h_2 are the respective heights to which the centers of mass rise. Combining Eqs. (1) and (2), we find

$$\frac{h_1}{h_2}=\frac{M_2(I_2+M_2r^2)}{M_1(I_1+M_1r^2)}. \tag{3}$$

The moments of inertia I_1 and I_2 can be calculated from the geometry of the pendulums; or if they are constructed such that $I_1\ll M_1r^2$ and $I_2\ll M_2r^2$, then Eq. (3) reduces

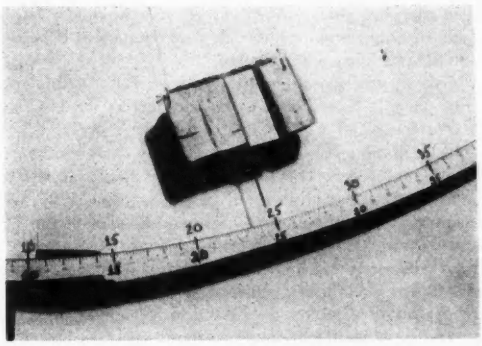

FIG. 2. A frame from movie film of a ballistic pendulum recoiling after being struck by a bullet. The numbers are retouched for clarity in reproduction; they show clearly in the original.

to

$$h_1/h_2 = M_2^2/M_1^2. \quad (4)$$

The student may use his data to verify this prediction.

One may also obtain the initial kinetic energies from Eq. (2), and thus study the sharing of energy in recoil. Agreement with expected results, to within about one percent, is obtained.

Another laboratory study to which motion picture photography lends itself is the measurement of rifle muzzle velocities with the ballistic pendulum. Although keenly interesting to most students, this experiment is often considered too dangerous for class work; the probability of a serious accident is never negligible. However, if the experiment is photographed beforehand by the instructor, the student may study the film in safety. Figure 2 is a sample frame taken from a 64-frame-per-second photograph of a ballistic pendulum. The deflection of the block is measured in degrees, the numbers being read from the pictures, and the height of rise calculated from the deflection and the length of the pendulum.

Other possibilities in dynamics suited to photography suggest themselves, such as falling bodies and billiard-ball collisions.

Some readers may believe that this method of laboratory study denies to the student the freedom of manipulating the apparatus. However, this disadvantage is minimized by having the student perform all operations except the actual photography (and, in the case of the ballistic pendulum, the actual firing). The disadvantage is further balanced by the improvement in accuracy which is gained when mechanical restrictions on the movement of the apparatus are removed.

[1] Lemon and Ference, *Analytical experimental physics* (University of Chicago Press, 1946).

AMERICAN JOURNAL OF PHYSICS VOLUME 17, NUMBER 8 NOVEMBER, 1949

Letters to the Editor

The Demonstration of Nuclear Magnetic Resonance

IT does not seem to have been generally recognized that the demonstration of nuclear magnetic resonance requires only quite simple apparatus and should be within the scope of the average college physics department. In spite of the recent development of the subject there is a widespread recognition of its importance as a research technique but as yet there seems to be little realization that explanation of the basic principles coupled with an actual demonstration could form a useful addition to many undergraduate physics courses.

The possible importance of the subject from a didactic point of view has been emphasized by Dr. K. K. Darrow in an address in which he reviewed the experimental work done to date and there is no doubt of the interest aroused when the phenomena are displayed and explained. In the case of proton resonance no elaborate equipment is needed to make possible such a display.

Of the apparatus required the magnet itself is the major component. It should have a field strength of not less than 4000 oersteds and a gap width of 2 cm or more with pole pieces large enough to ensure homogeneity over a volume of the order of 1 cm³. If such a magnet is available the remainder of the equipment may be assembled without difficulty. A 300-volt power supply and an oscilloscope are required and a small regenerative detector and amplifier circuit will need to be built. Constructional details for this are identical with those given elsewhere[1] for a magnetic field strength meter based on the same principle. They need not be repeated here. Some simplification is possible, however, for the present purpose since neither portability nor a broad range of application is required. The probe and amplifier described in the paper referred to may thus be incorporated into a single chassis and the so-called "wobbling" coils may be wound on the magnet pole pieces in any arrangement which will give a suitable wobbling amplitude. Also, the only sample coil required will be the one appropriate for the available magnetic field strength.

In summary: it has been pointed out that with the construction of a small electronic circuit and the use of standard laboratory equipment it is possible to display proton magnetic resonance.

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[1] Hopkins, *Rev. Sci. Inst.* **20**, 401 (1949).

Velocity of a Longitudinal Wave by an Elementary Method

ASSUME that a prismatic bar, of cross-sectional area A, density ρ and Young's modulus E, is subjected to a sudden compressive force uniformly distributed over the face BC (Fig. 1), causing a displacement $BB' = s$, and suppose that the disturbance is transmitted in an infinitesimal time t to a section GH, at a distance $BG = x$ with a velocity v, so that $x = vt$. If we call a the average acceleration of BC during the time t, $s = at^2/2$, whence $a = 2s/t^2$.

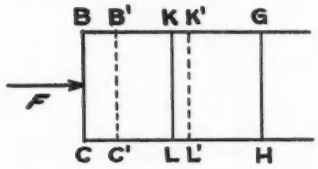

FIG. 1. Longitudinal wave traveling along a solid bar.

The portion $BGHC$ of the bar has the mass ρAx, is compressed by s, and the net force acting on it is $F=EAs/x$. If we do not suppose v to be infinite, x is infinitesimal, and we may consider the displacements BB', $\cdots$, KK', $\cdots$ of the sections BC, $\cdots$, KL, $\cdots$ as directly proportional to the distances BG, $\cdots$, KG, $\cdots$, respectively, while the displacement of section GH is zero.

Consider the portion $BGHC$ divided into n transverse layers of equal thickness and common mass μ, so that $n\mu=\rho Ax$; then, we may think of F as resolved into n forces acting on the n layers and displacing them with the accelerations a, $\cdots$, a_k, $\cdots$, 0 during the time t. These accelerations are directly proportional to the respective displacements; they form an arithmetical progression and their sum is $na/2$, while the sum of the components of F is $\mu na/2=\rho Axa/2=\rho Axs/t^2$, and we can write $F=\rho Axs/t^2$.

Equalizing both expressions for F, $\rho Axs/t^2=EAs/x$, or $x^2/t^2=E/\rho$. The quantity in the left side of this equation is v^2, and the formula $v=(E/\rho)^{\frac{1}{2}}$ is readily obtained.

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Cuba

An Instructional Technique for the General Physics Laboratory

AN instructional technique which has been remarkably successful in raising the level of student performance has been in use in the general physics laboratory at Temple University for a period of over twenty years and has not, to my knowledge, been described elsewhere. The technique involves the use, in a rather unconventional way, of undergraduate majors as laboratory assistants.

General physics laboratory sections at Temple run to about eighteen students in size, and meet once a week for a period of three hours. To each section of this size there are assigned, in addition to the regular staff member in charge, two undergraduate laboratory assistants so that the staff-to-student ratio in the laboratory is about 1 to 6.

Experiments are assigned on an individual basis, each student being responsible for obtaining his assignment from the instructor in charge a week before the date on which the experiment is to be performed. No attempt is made to have all students in a section perform the same experiment during a given laboratory period nor is there any effort toward maintaining the same pace for all students. Thus each student works at his own pace, being motivated so far as speed is concerned only by the requirement that a certain minimum number of experiments be completed during the semester.

The uniqueness of the Temple scheme lies in the use made of the undergraduate laboratory assistants. These assistants, who are junior and senior majors in the department, spend their time quizzing the students, usually in groups of two or three at the blackboard, about the underlying principles and experimental techniques involved in the experiments on which the students are working. The questioning is generally of the Socratic type, the assistant attempting to draw out of the students a connected account of the particular experiment.

The members of the general physics class are told, of course, to expect these oral laboratory quizzes but what they do not know is when the quiz will occur in any particular case. The student may be called to the blackboard at the beginning of the laboratory period before he has even had an opportunity to inspect his equipment or he may be called up when he is part way through the experiment or after he has completed it. Depending upon his preparation and his ability to answer the assistant's questions, the quiz may be short, lasting only fifteen or twenty minutes, or it may take up the full three-hour laboratory period. After one experience with a three-hour quiz on an experiment or, if he is wise, before such an experience, the student will acquire the habit of coming to the laboratory thoroughly prepared on the experiment to which he has been assigned.

The advantage of this scheme in increasing the effectiveness of the general physics laboratory is fairly evident. The student gets much more out of his experimental work because he comes to the laboratory properly prepared and does not, therefore, perform his experiments in the more common cookbook style of alternating a reading of the instructions with an adjustment of some control or the reading of a scale. The advantages accruing to the departmental majors who serve as laboratory assistants are, however, equally important. They feel that they are part of the instructional team of the department and thus acquire, early in their work, a professional awareness as physicists. An unusually large number of those who serve as undergraduate laboratory assistants continue their work in graduate school and eventually receive doctor's degrees.

The number of undergraduate assistants required at Temple to implement the scheme described is about equal to the number of departmental majors so that all majors who desire to work in this way may obtain employment. In general, each assistant devotes nine hours a week to this work and receives pay of approximately a dollar and forty cents per hour.

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Airy's Theorem and the Improvement of Clocks

THE article by Mr. Gaehr on Airy's theorem in the *American Journal of Physics* [16, 336 (1948)] contains some slips which, in the interests of accuracy, call for correction.

The author of the Theorem is referred to on page 336 as *Lord* Airy. Airy was never a peer, but died in his 90th year as *Sir* George Airy.

Lord Grimthorpe, who was a peer, published the last edition of his treatise in 1903, not in 1872 as stated on page 336.

The statements about Harrison's chronometer, beginning at the foot of page 337, are also wide of the mark. The escapement is described and illustrated in

Britten's *Old Clocks and Watches* (Spon, 6 ed.), page 375. It was a crown wheel and verge escapement exactly as shown in Gaehr's Fig. 1 and without question far inferior to the detached escapement invented by Mudge.

On page 341, Mr. Gaehr uses a circle diagram to derive the change in time due to the impulse—in other words, the escapement error. I cannot quarrel with the general method, since I have published a derivation of Airy's integral by a similar process (*Science of Clocks and Watches*, Pitman). Mr. Gaehr's result does not agree with Airy; and in particular none of his expressions for ΔT on page 341 has the dimensions of a time as it should.

The failure of Huygens' cycloidal pendulum is ascribed by Mr. Gaehr to its introducing escapement errors. The ground for this assertion is not clear, since Mr. Gaehr's expression for the escapement error has no term which a cycloidal pendulum could affect; and furthermore, Airy explicitly stated that his results applied to a cycloidal pendulum only.

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Airy's Theorem and the Improvement of Clocks: a Reply

SINCE the appearance in this magazine last September of my article "Airy's Theorem and the Improvement in Clocks," a number of letters pointing out errors have come to me, among them the letter of Mr. A. L. Rawlings above. I should be grateful if, in the interests of accuracy, the corrections listed below be published.

As may be inferred from my article, I did not have access to many books on the theory of clocks, although I have hunted through four large libraries. None of them included the excellent and recent book on *Science of Clocks and Watches* by A. L. Rawlings (Pitman, rev. ed., 1948), which should therefore have been included in the list of recommended references. Mr. Rawlings writes that he has there given a demonstration of Airy's equation by a method similar to mine.

Lord Grimthorpe's latest edition appeared in 1903, instead of in 1872.

Lord Airy should read *Sir* George Airy.

Page 338, first sentence of first column: the escapement of Harrison's chronometer was not better than Mudge's.

Page 339, second column, line 10, *et seq.*: The whole duration of an impulse was about *one-fourth* of the swing from left to right. . . . the work of each impulse was calculated to be 25.6 ergs. In the 13th line of the first column of page 340, change 2.5 ergs to 20.8 ergs.

Near the bottom of the first column of page 341, and first line of the second column, the expressions θ_1/π, θ_2/π, $(\theta_1-\theta_2)/\pi$ should all be multiplied by $T/2$. In the equations for ΔT, read $2\cdot\Delta T/T$.

In the second column of page 342, I wished to say that in eliminating one error, clock makers often introduced another, giving Huygen's cycloidal pendulum as an example. One writer stated that Huygen's pendulum failed because of escapement errors; another writer gave the friction of the cycloidal cheeks as the cause; and still another writer said that the bob was much too large to be considered as a point.

PAUL F. GAEHR

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An Interesting Exhibit

THE writer was recently asked to make up a group of exhibits for a college "open house" day. One of the exhibits which attracted considerable attention of both technical and nontechnical guests consisted of a fluid rotating at a speed of several revolutions per minute without a mechanical propellor. The fluid was propelled by the action of a vertical magnetic field on moving ions.

The exhibit is shown in Fig. 1. The field was provided by

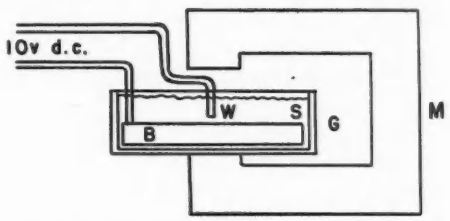

FIG. 1. A current-carrying electrolyte in a magnetic field.

a large magnetron permanent magnet *M*. A glass container *G*, approximately 15 cm in diameter and 5 cm deep, was placed between its poles and contained a quantity of concentrated copper sulfate solution *S*. A copper wire *W* was suspended from the upper pole of the magnet and allowed to dip down into the container. A thin copper band *B* was inserted along the inner periphery of the container and a potential of 10 volts was applied between these electrodes. The direction of the current was reversed from time to time, which, of course, caused the direction of rotation of the fluid to change. Two copper electrodes are used for simplicity and to avoid the occurrence of chemical changes in the system.

It is hoped by the author that this presentation will be of use to some poor soul faced with the dilemma of providing an entertaining public exhibit, while simultaneously satisfying technical people that he is not merely deceiving the public with spectacular nonsense.

J. L. RYERSON

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Evansville, Indiana

Can We Fly to the Moon: Errata

SEVERAL errors appeared in the paper by Dr. Himpan and the writer entitled "Can We Fly to the Moon."[1] The list of corrections is as follows:

(1) Page 257, Eq. (2) should read

$$h_c = w_c t_c - w_c(G_a - T/\dot{G}) \ln(G_a/G_a - T) - \tfrac{1}{2} g_x t_c^2.$$

(2) Page 258, left-hand column, lines 32 and 33 should read

$$G_a = 5 \times 10^3 \times 5 \times 10^3 N_c = \cdots$$

(3) Page 259, left-hand column, lines 14 and 15 should read

$$v_{ei} = 3540 \text{ m/sec.}$$

(4) Page 259, left-hand column, line 32 should be a trifle more than the 5.07×10^7 kg m that is . . .

(5) Page 260, left-hand column, the line preceding Eq. (11) should read
By integration of Eq. (10) it is found . . .

(6) Page 260, Fig. 4:
Values of the ordinate $\dot{G}$ must be multiplied by 10^{-6}. Thus the ordinate-values shown in ascending order are: 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, and 10^{2}.

(7) Page 261, right-hand column, line 9 of text should read
with an equivalent top speed of . . .

(8) Page 262, left-hand column, line 5 of item (b) should read
The increase is proportional to $(H_T)^{\frac{1}{2}}$.

R. REICHEL

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St. Cloud, France

[1] Himpan and Reichel, *Am. J. Physics* **17**, 251 (1949).

The Dimensions of Angular Displacement and Torque

IN a recent article in this journal by P. Moon and D. E. Spencer[1] it was recommended that the linear dimension be split into two in order to provide a 1:1 relationship between concept and dimension in the case of angular displacement, moment of a force, and torque. I would like to suggest that the assignment of primary dimension to angular displacement (1) is more logical, and (2) is necessary in order that the dimension of a physical quantity yield correct information about it.

Concerning item (1) it is generally contended that angular displacement is a numeric since angle is the ratio of two lengths. But the relation $\theta = s/r$ is a special case of the more general one $\theta_1/\theta_2 = s_1/s_2$, or $\theta = ks/r$ where k is a proportionality constant and θ may be measured in any unit. It is θ/k which is dimensionless, not θ. To assume that choosing θ in radians makes k dimensionless, then to use this assumption to prove that the resulting relation implies that θ is dimensionless, is circular reasoning. If a linear unit of such length were chosen that the gravitation constant G would be numerically 1, and if it were then assumed that this made it a numeric, then the resulting relation between l, m, and t implied by the equation $F = m_1 m_2/d^2$ would destroy most of the utility of dimension theory in mechanics.

Another argument for assuming the radian to be dimensionless is that the Maclaurin's expansion of sin θ necessitates that θ be a numeric. In Eshbach's *Handbook of Engineering Fundamentals*, page 3-37, is the following statement: "Functional forms (like trigonometric or exponential functions) can have only numerical arguments; sin E, where E means energy, or voltage, has no sense because it violates dimensional homogeneity. This can be seen by expanding the function into the equivalent power series,

$$\sin E = E/1! - E^3/3! + E^5/5! - \cdots,$$

where each term has different dimensions." This argument is not valid. The n^{th} term of the Maclaurin expansion of a function is $f^{n-1}(0)\theta^{n-1}/(n-1)!$ For any function the derivative $f^{(n-1)}(\theta) = d^{n-1}f(\theta)/d\theta^{n-1}$ has the dimension of $f(\theta)/\theta^{n-1}$ for any value of θ, and hence the general term has the dimension of $[d^{n-1}f(\theta)/d\theta^{n-1}]\theta^{n-1}$, that is, of $f(\theta)$; this is independent of n, so the series is dimensionally homogeneous. A similar statement holds for the Taylor expansion.

Concerning item (2), I would delay the discussion of the moment of a force until work and potential and kinetic energy are defined. Then a qualitative definition of the term can be stated, as usual, as the ability of a force to produce rotation of a body about an axis. A quantitative definition can then be arrived at by considering two parallel but oppositely directed forces F_1 and F_2 which act perpendicularly on a bar of constant mass (concentrated at points or distributed) which is free to turn about one end, the points of application of the forces being distant r_1 and r_2 units, respectively, from the axis. Then, instead of giving experiment as the basis for the relation $F_1 r_1 = F_2 r_2$ (as is the practice in texts), the result can be obtained from the law of the conservation of energy, thus reducing by one the number of assumptions in elementary mechanics.

Let XY be a line in the plane of the forces and passing through the axis, and θ the smallest positive angle through which the bar must be rotated in the direction of F_1 to bring it into coincidence with line XY. Suppose the bar is in equilibrium and the magnitudes of the forces are constant. Then F_1 alone is *potentially* capable of rotating the bar through the angle θ (the direction of the force remaining perpendicular to the bar) thereby adding an amount of kinetic energy which must equal $F_1(r_1\theta/K)$ regardless of the masses involved or the final velocity. In like manner the potential energy of F_2 with respect to the line XY is $-F_2(r_2\theta/K)$. Since the bar is assumed to be motionless with respect to the line XY, its kinetic energy is zero, and from the conservation of energy the algebraic sum of the potential energies is zero; i.e., $F_1 r_1\theta/K - F_2 r_2\theta/K = 0$, or $F_1 r_1 = F_2 r_2$. It should be emphasized that the direction of the distance through which the force acts at any instant is the direction of the force, and not that of the normal to the force; that is, the quantity $F_1 r_1 \theta/K$ is work, and is measured exactly as work is always measured. We now take as our measure of the moment of either force about the axis the energy which it (alone) is potentially capable of adding to the system per unit angular displacement. In any system of units in which the radian is the unit of angle, the moment M of a force is equal to Fr energy units per radian. In the gravitational system the unit is ft-lb per radian. The writing of the energy unit as lb-ft is un-

necessary and misleading, and the omission of the angular displacement dimension makes the meaning vague.

The extension of the definition to a body in rotation readily leads to the conclusion that torque is the rate of change of kinetic energy per unit change in angular displacement ($=d(\frac{1}{2}I\omega^2)/d\theta$). For constant masses torque and force are thus the rates of change of rotational and translational kinetic energy, respectively, the former with respect to unit angular displacement, and the latter with respect to unit linear displacement ($=d(\frac{1}{2}mv^2)/ds$).

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[1] Moon and Spencer, *Am. J. Physics* **17**, 171–177 (1949).

AMERICAN JOURNAL OF PHYSICS VOLUME 17, NUMBER 8 NOVEMBER, 1949

ANNOUNCEMENTS AND NEWS

Book Reviews

Principles of Electricity and Magnetism. Second Ed., GAYLORD P. HARNWELL. Pp. 670, Figs. 398, 15×23 cm. McGraw-Hill Book Company, Inc., New York, 1948. Price $6.00.

The first edition of this book appeared in 1938, and was reviewed in this *Journal* (**7**, 203 (1939)). The outline of Professor Harnwell's text remains essentially unchanged in the new edition. However, the significant developments during the past ten years in the fields of atomic and nuclear physics and in the use of electromagnetic waves have influenced the emphasis on and the selection of topics in this new edition, e.g., the accounts of electronics and of radiation have been considerably expanded in the direction of higher frequencies, and a brief account of generators at higher frequencies and of resonant cavities and wave guides is now included, as well as some material on semiconductors.

Throughout the book, the treatment is developed from the point of view of the physicist; the fundamental aspects are emphasized, and the experimental basis of the considerations leading to Maxwell's equations are clearly displayed. The student using this text not only will learn about electricity, but will get an integrated picture of a great part of modern physics. Many applications from neighboring fields are given; those bearing on topics of current interest are stressed somewhat, as for example accelerators or the nuclear induction experiments.

The text starts with electrostatics; vector analysis is freely used in the formulation of the general concepts, and complex variable methods are used in the discussion of special problems. The discussion of the electromagnetic effects is based on Ampère's experiments. The electrical charge is considered as the fundamental electrical entity here as well as in the subsequent treatment of magnetism. Magnetic poles are used only later, as an aid in calculation.

The chapter on nonohmic circuit elements is greatly enlarged and contains material not found in other texts. The chapters on a.c. and vacuum tube circuits, and on coupled circuits and lines, contain a wide selection of topics; emphasis is placed on an understanding of phenomena, rather than the description of devices or their functioning. A concise chapter on the basic types of electrical machinery is also included.

A summary of vector analysis and of frequently used material on differential equations is included in an appendix. The mks system of units is used principally, but the older systems of units are mentioned and used occasionally. The text contains a wealth of problems, many of them new, and most of them having real physical meaning, like the problems in magnetism where pole pieces used in molecular beam experiments are discussed.

The preface states that the text is intended for students having a sound preparation in general physics and in mathematics up to and including differential equations. Some previous acquaintance with topics usually covered in advanced calculus, seems also indicated, since div **grad**ϕ is used on p. 30 and $\log(x+iy)$ on p. 40. This may make the book a bit hard for many undergraduate students.

Notwithstanding the use of mathematical tools, where they are a help in clarifying an argument, the physical problems and concepts and the experimental physicists approach always remain preeminent.

In view of the many valuable features of Professor Harnwell's book, and of its whole approach to intermediate teaching, it should be underscored that it could also serve in courses for students lacking a certain degree of mathematical maturity. This is possible because the more advanced mathematical tools are actually used only in the more theoretical chapters on electrostatics, electromagnetism, and radiation. For example, the more qualitative chapters on dielectrics and magnetic materials, or the chapter on photoelectric effect and related phenomena, with its discussion of thermionics and of semiconductors, could provide an excellent source of collateral reading material for any type of course in electricity. This book might prove to be an excellent text for an advanced physics course serving primarily the needs of engineering students. It certainly should provide a source of stimulating reading for any beginning physicists or their teachers.

F. T. ADLER
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Elementary Photography. Second Ed., GILFORD G. QUARLES. Pp. 345, Figs. 42, Plates 78 (2 in color), 6¼×9¼ in. McGraw-Hill Book Company, Inc., New York, 1948. Price $4.50.

The general nature of this interesting elementary book on photography is very well described by the author's remarks in the preface to the first edition. It is designed to be intermediate in character between very elementary books suitable for high school use and other quite advanced ones too technical for general reading. It was written primarily for use as a textbook for students without the background of physics and chemistry, but also with the desire to make it a useful tool for the average amateur photographer, or beginner. The author has succeeded very well in his project.

The range of subject matter covered is unusual. To quote, the author "knows of no other book on the general subject (photography) which treats of pictorial composition, projection control, picture making with paper negatives, toning, hand coloring and natural color in such a manner as to be usable by any serious worker, regardless of previous training."

In the preface to the new (second) edition the author states that the revision has not been made in order to incorporate new material, but rather to improve its "teachability" by making certain additions and rearrangements. The principal changes are the addition of study questions at the end of each chapter, and a rearrangement of the chapters. The study questions are quite numerous, and well chosen, and should direct the efforts of the reader in a very profitable manner. The rearrangement is typified by the fact that Chapter II is a short discussion of the most elementary optics of the camera, further discussion following in later chapters, and is followed immediately by "Film Sensitivity and Exposure," Chapter III, and by "Development of Negatives," Chapter IV, the main idea being to lead the student as soon as possible into the actual photographic process.

The subject matter which of necessity appears in any book on photography, however elementary, is very well presented. The discussion is always simple, complete enough to satisfy any beginner, never stilted. Chapters VII and VIII complete the discussion of optics, and include the aberrations of lenses and such items as the view finder and the range finder. Chapter X deals with the color response of the various common emulsion types and the use of correction filters and contrast filters. Chapter XV gives the elements of projection printing, and Chapter XVIII describes the art of "finishing" the print.

Certain chapters are unusual in an elementary treatise on photography. One usually hunts for such material in separate books. Here one finds a very nice discussion of "Composition," Chapter XVI, the fine points of enlarging in "Projection Control," Chapter XVII, and "Picture Making with Paper Negatives," Chapter XIX. "Natural Color Photography," Chapter XXII, however, receives a rather sketchy treatment.

The book ends with a suitable bibliography, Appendix A, and a formulary, Appendix B, which contains the essential and standard formulae indispensable to any photographer, amateur or otherwise, who really practices the art. After reading the book, one has the feeling that here is a job well done.

I. C. CORNOG
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The Structure of Matter. FRANCIS OWEN RICE AND EDWARD TELLER. Pp. 361+xiii, Figs. 69, Tables 13, 6×9¼ in. John Wiley & Sons, Inc., New York, 1949. Price $5.00.

This book, the first of Wiley's Structure of Matter Series, is a valuable addition to the already available books on the application of quantum theory to the interpretation of the behavior of matter. The reviewer's very favorable reaction is based on the style of the book, rather than on the inclusion of any new material. The book makes no pretense of adding to the known facts, but it does present these facts in a way that should appeal to a large audience, surely eager for such a style of exposition.

Many textbooks and reference books require quite a lot of between-the-lines reading and interpretation. One of the main functions of a teacher is to simplify this task for the student; and one of the chief advantages of taking a formal course in a subject, rather than depending on self-study, is this service rendered by the teacher. Much of this is accomplished in the classroom by what the reviewer likes to call the "hand-waving" technique. One talks in a general way and tries to create mental images that illuminate and clarify the subject, and make it more tangible and friendly and less formal and obscure. *The Structure of Matter* is one of the rare books that offer this service to the reader of other texts and reference books dealing with similar material. In addition, it presents a rather extensive set of phenomena associated with the properties of matter, all presented with clear qualitative discussions that give the book its particular quality. It thus serves as a useful source of material, as well as an interpreter of theories. The authors state as the purpose of the book the wish "to acquaint the reader with the scope of the phenomena that can be explained with the help of quantum mechanics." In attempting to do this, most of the formal mathematical analysis is omitted and the qualitative results of the theory alone are presented. One may, at first thought, regret the omission of the mathematical details; but, as one becomes better acquainted with the book, the advantages become evident.

An important technique of thinking about physical phenomena is the use of reasonable estimates to suggest the behavior to be expected. *The Structure of Matter* makes frequent use of this technique, reminding the reader of its power. Unfortunately, the first example gives the precise rather than an approximate answer! This result occurs in the calculation (p. 7) of the energy of the ground state of the hydrogen atom. The uncertainty relation in the form $\Delta p \Delta q \sim h/2\pi$ is used in a clever way to give the result. The reviewer objects to the combination of an accurate answer coupled with a lack of emphasis on the fact that a mere estimate is being made. If the authors had used $\Delta p \Delta q$

$\sim h/4\pi$, a common form, the result would have differed from the true value, but would have given its order of magnitude. The approximation would have emphasized the limitations as well as the advantages of the method. It is interesting to note that a similar calculation for the ground state of the linear harmonic oscillator, based on the form $\Delta p \Delta q \sim h/2\pi$, gives $h\nu_0$, while the form $\Delta p \Delta q \sim h/4\pi$ leads to $\frac{1}{2}h\nu_0$, the correct value. In later applications the authors make the approximate nature of their results more apparent.

The general topics included are suggested by the chapter headings: Introduction (an introduction to quantum principles), The Hydrogen Atom, The Periodic System, Motion and Position of Nuclei in Molecules, Atoms and Molecules in Electric Fields, Van der Waals Attraction Forces, The Chemical Bond, Forces in the Solid State, Magnetic Properties of Matter, Molecular Vibrations, Electronic Spectra, Nuclear Chemistry, State of Matter in Stars. This list shows that the contents include the bulk of the results of the "classical" (that is, the pre-1940) quantum theory. An amply diverse lot of examples is given in each chapter. A specialist in a field may regret the omission of certain examples and may find minor errors or indirections. For example, this reviewer was a bit unhappy about several aspects of the treatment of atomic spectra, and would have preferred a closer adherence to the conventional notation. The authors do not use a subscript to designate the value of J, and they use u and g to label odd and even states, contrary to the more common use of the symbol ° for odd states and omission of any symbol for even states.

The use of diagrams has not been exploited in this book as fully as it might have been. While it is difficult to conceive a really adequate graphical scheme conveying to the reader the spatial form of wave-functions, and while there is need for the development of such a system, this reviewer feels that Rice and Teller have not improved over other schemes. One final objection is to the complete lack of references to original and comprehensive treatments of the various topics. Although the inclusion of such references might have required too many footnotes or too long a bibliography, this book should arouse such interest in the various examples, that the reader will almost certainly want to carry on his study and to refer to the more complete treatments. A bibliography would then be a great help.

These criticisms are minor and should not mask the very many worthy features of the book, which far outweight such picayune objections. *The Structure of Matter* should be a useful book for nearly any physicist, regardless of his special field. It will remind him of the facts he has learned and may have forgotten; and it will probably introduce him to other fields of which he is unaware because removed from his main interests.

Although students with little training in quantum mechanics can get a great deal of good out of this book, it is really addressed to those who are already familiar with quantum mechanical techniques. Others may not be able to appreciate fully many of the points that are discussed.

The Structure of Matter is basically sound, with only a few minor deviations from customary views. It offers a new approach. It is a source of much useful information. It should prove to be a book that is referred to constantly.

A. T. GOBLE
Union College

Elektrizitätslehre. R. W. POHL. Pp. 301, Figs. 497, 17×25 cm. Springer Verlag, Berlin-Göttingen-Heidelberg, 1949.

The review of a German textbook in an American journal is justified only when it contains certain outstanding features, or gives opportunity to general reflections on educational problems which are under discussion on this side of the Atlantic. Both can be said about Professor Pohl's *Electricity*, which forms the second volume of an *Introduction to Physics*, the first volume dealing with mechanics and the third with light.

Pohl's *Electricity* was published in an English version in 1930, and issued several times thereafter (Blackie and Son Ltd., London and Glasgow). A comparison of the new book with the English translation shows considerable changes and in some instances definite improvements.

The most attractive feature of the book has always been the great number of excellent figures, representing shadow projection pictures of instruments and apparatus as they are shown in Professor Pohl's classes for beginners at Göttingen university. In central European universities it is the common practice in the first physics course to produce ample experimental evidence through demonstration experiments. These courses are called "experimental physics," and the name was often understood as an obligation to proceed from experiment to experiment. In this way the lecture course became more and more a continuous show, and the intellectual level of instruction necessarily suffered. Lack of time hindered the lecturer to give theoretical analytical demonstrations, or to point out the historical development of physics. The wealth of experiments in Professor Pohl's lectures, I was told, was truly appreciated only by advanced students, who frequently returned to his classes on account of the excellent demonstrations of physical phenomena. The success of Pohl's books was also greater among instructors and teachers in secondary schools (gymnasia), than among university students. Books and demonstration technique were also adapted for more elementary school physics courses.

The new edition of the *Elektrizitätslehre* corrects to a certain degree the fault of neglecting mathematical demonstrations. Historical discussions are now more numerous, but still appear somewhat superimposed on the subject-matter. The book should now have greater usefulness for the younger student, but will still appeal more to the teacher, I believe. It is paradoxical that Pohl's book can be considered as highly didactical, although it is not really pedagogical.

At the time of its first publication Pohl's *Electricity* was outstanding through two new features: the use of the practical electrical units throughout, and the change in the sequence of experimental demonstrations, on which Pohl prides himself particularly: "The starting point of our experiments are not rubbed amber and bits of paper, but

chemical sources of electricity as dry batteries and storage cells. . . . We measure *electrical* quantities in *electrical* units, as multiples of a unit-current (amp) and of a unit-tension (volt)." Pohl considers electric current and voltage as terms with which the student is familiar, and uses ammeters and voltmeters from the beginning in the same way as one uses clocks and watches in mechanical experiments, i.e., without a previous investigation of their construction.

For the consistent use of practical (now international) electrical units, and the rational system, Pohl's book did pioneer work. He makes reference to the electrostatic and the electromagnetic systems only in passing. There is little doubt that for the engineer and applied physicist the use of practical electrical units of the meter-kilogram-second system (joule, watt, volt, ampere, ohm, coulomb, henry, farad, weber (=volt sec)) has great advantages. The two additional vacuum-constants, permittivity $\epsilon_0 = 8.86 \times 10^{-12}$ and permeability $\mu_0 = 1.256 \times 10^{-6}$, cannot be considered to present serious difficulties. The appearance of the factor $1/4\pi$ in Coulomb's laws is compensated by the disappearance of this factor from other equations, e.g., for the capacitance of the plate condenser, and the energy of the electric and magnetic fields.

However, from the didactic standpoint arises the question that we have to explain why the numerical values of ϵ_0 and μ_0 can be "theoretically" expressed as $1/(4\pi \times 9 \times 10^{12})$ and $4\pi \times 10^{-7}$. Had the practical units been introduced without any reference to the absolute units, and had the constants arbitrary numerical values, there would be no difficulty. As it is, the thoughtful student wants an explicit explanation, which he would not find in Pohl's book. There is no American or English physics textbook, say on the sophomore level, which attempts an exposition of this part of an electricity course in a comprehensive and comprehensible way. To use different units and systems in different parts, and then to *connect* them in an appendix or a table, as is often done, is a sure way to bury the problem.

One third of Pohl's book is devoted to the principles of electric conduction, in all three states of matter; valuable material is included in this section which in America would be mostly treated in lectures on modern physics. There are also special chapters on Radioactivity, on Electric Waves, and on "Relativity as a Principle of Experience." The book can well be called a complete work, which shows Professor Pohl's ingenuity as an experimentalist, and also his experience as a teacher by the lucid explanations of questions of detail.

There are some noticeably eccentric features in the book. Pohl does not use the name newton for the mks unit of force, but speaks about *Grossdyn* (Great-dyne). Instead of joule he writes for the energy unit *Grossdyn-meter*. In the English translation from an earlier German edition the force unit is referred to as "the nameless force unit," but the translator of a new English version would be well advised to conform with English-American and international usage. This would also apply to the translation of the German *Spannung* either by tension or voltage, but not by pressure.

More odd even than the apparent avoidance of newton and joule (but not of watt), is the introduction of new names for improvised units which are used for a certain demonstration—here insignificant—at the end of the book. Pohl gives them everyday names, namely Fritz for Widerstand (resistance), Max for Spannung (voltage) and Moritz for Strom (current). With the choice of the names *Max* and *Moritz* Professor Pohl refers to the classic German comic children's book; he is evidently joking, and I think everybody can well see the humor of it. However, if we are allowed to recognize the great-in-the-small, does not Professor Pohl's attitude in respect to the naming of physical units in a scientific text show signs of a fallacious isolationism and parochialism in contemporary German scientific education?

OTTO BLÜH
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AMERICAN JOURNAL OF PHYSICS VOLUME 17, NUMBER 8 NOVEMBER, 1949

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [*Am. J. Physics* 17, 456 (1949)].

Baber, Stanley (*J*), 5002 S. "J" St., Tacoma 8, Wash.
Badgley, Ralph Emerson, Eastern Oregon College of Education, La Grande, Ore.
Beers, Yardley, 4 Ploughman's Bush, Riverdale 63, N. Y.
Bolen, Max C., Physics Department, The James Millikin University, Decatur, Ill.
Bragg, Robert H. (*J*), 5623 Wabash Ave., Chicago 37, Ill.
Burns, Jay III, 1712 W. Estes Ave., Chicago 26, Ill.
Carlson, Henry Arthur (*J*), Rte No. 1, Blairstown, N. J.
Causey, Robert L. (*J*), 936 Covington Ave., Bowling Green, Ky.
Coe, John E., 2024 Sunnyside Ave., Chicago 25, Ill.
Coyle, Thomas (*J*), 910 Franko St., Throop, Pa.
Darlington, Lillian E. (*J*), 6630 S. Peoria St., Chicago 21, Ill.
Desjardins, Lucien Henri, 52 Assumption Ave., Worcester 6, Mass.
Ellison, Robert William (*J*), 717 S. 20 St., Lafayette, Ind.
Forrester, Alvin Theodore, University of Southern California, Los Angeles 7, Calif.
Garr, Carl Robert (*J*), 127 Pittsburgh St., Ravenna, O.
Goldstein, Herbert, 39A Lee St., Cambridge 39, Mass.

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McCormick, Thomas Herbert, Box 5, Tuskegee Institute, Ala.
McHenry, James R. (*J*), 201 West Wood St., Norristown, Pa.
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Science Courses in General Education

College science courses designed for general education exist in more than half of America's four-year colleges and universities according to a survey recently completed by Robert A. Bullington of MacMurray College, Jacksonville, Illinois. The study, which formed the basis of a Ph.D. dissertation at Northwestern University, was carried out under the sponsorship of the Cooperative Committee on Science and Mathematics Teaching of the American Association for the Advancement of Science.

Information concerning science offerings obtained from 720 institutions revealed that 293 or 41 percent offer no science course specifically designed for students who are not majoring in science while 427 or 59 percent offer general education science courses of various types.

Detailed information concerning the nature and objectives of 150 of these general education courses in science was obtained by means of questionnaires and by inspection of syllabi, manuals, reading lists, examinations and published articles, especially those in *Science in General Education*, edited by Earl J. McGrath. In addition, 17 schools were visited and information obtained firsthand about 28 courses.

The study brought to light some interesting facts concerning the teaching approach used in general education science courses. The subjectmatter survey approach is rapidly losing ground and is employed in fewer than one-third of the courses offered. In more than two-thirds of the courses the teaching approach involves an intensive study of selected units or problems or the historical development of principles and concepts.

A number of trends are indicated by the study. One is in the direction away from the free election of general education science courses and toward the definite prescription of such courses; another is toward the single-subject course as opposed to the type in which material from a number of sciences is treated in the same course.

There is no unanimity of opinion on the objectives to be achieved in these courses though there appears to be increasing emphasis on developing an appreciation of the scientific method and understanding of principles and a decreasing emphasis on wide coverage of facts.—B.B.W.